A COMBINATORIAL APPROACH TO OPTIMIZING THE PLACEMENT OF IRREGULARLY SHAPED ELEMENTS ON TWO-DIMENSIONAL AND THREE-DIMENSIONAL SWITCHING FIELDS WITH COMPLEX TOPOLOGY

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This paper presents a novel optimization method for placing non-standard configuration elements on two-dimensional and three-dimensional interconnection fields with complex topologies. A combinatorial analog of the Gauss-Seidel method has been developed, adapted for efficient handling of components with irregular shapes and varying dimensions. The algorithm accounts for specific characteristics of the interconnection field, including the presence of forbidden zones and structural heterogeneity. To increase the probability of finding the global optimum, a multi-start procedure has been applied. A software implementation has been created with modules for placement optimization and result visualization. The approach's effectiveness has been verified on original test problems of the "multidimensional snake" type. Examples for two-dimensional and three-dimensional fields with elements of various sizes and geometric constraints are considered. The results demonstrate the method's adaptability to different placement conditions for non-standard shaped elements. Quasi-optimal placement with an error of 0.055% was achieved in 1010 iterations, with computation times of 5 minutes for two-dimensional and 10 minutes for three-dimensional problems. The method is applicable in integrated circuit design, electrical equipment layout, and optimization of equipment placement in production systems. Quantitative assessments of the algorithm's efficiency for fields of varying complexity are presented. The implementation features for components with non-trivial geometry and interconnection fields of non-standard topology are analyzed. The advantages of the proposed approach compared to classical optimization methods for placing elements of different sizes are identified. Directions for further research are defined, including adaptation to specific requirements of various application areas and development of parallel computing strategies for large-scale problems with irregularly shaped elements on multidimensional interconnection fields with complex topologies.

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Introduction

Today, with the electronics and engineering industries advancing at a breakneck speed, the issue of optimal location of technology and other objects is critically important. A good component layout in electronic circuits, equipment in manufacturing plants, and other elements in various technical systems directly affects their performance, energy efficiency, and economic feasibility.

This problem is *NP*-hard and, as such, belongs to the class of combinatorial optimization problems. Even for a small quantity of components, the number of potential options makes an exhaustive search impractical. For example, just 16 elements have about $2 \cdot 10^{13}$ possible placement options. For 25 elements, this number is already on the order of 10^{25}. This fact necessitates the elaboration of approximation algorithms that would be sufficiently effective to find near-optimal solutions in a feasible period [1].

Problems of location involving items of different dimensions and irregular forms are particularly difficult when the most complex topological objects are considered in addition to cells of the simplest forms. This is a common issue in the design of electronic devices, where components may have varying dimensions and in solving optimization problems for the layout of industrial plants (Fig. 1-3) [2-5].

The current placement problem is solved by such approaches as genetic algorithms, particle swarm optimization, and other metaheuristics; however, they do not guarantee the quality of the solution or are of high computational complexity. This underlines the relevance of developing new effective methods and algorithms in optimizing element placement, with a variety of sizes and shapes.

Placement optimization problems in electronics

In modern microelectronics, the problem of optimizing the placement of elements with diverse sizes is becoming increasingly relevant due to the constant complication of integrated circuit design and the tightening of requirements for their characteristics [4, 6]. In microelectronics, the factor of availability of effective computer-aided design (CAD) tools is gaining ever greater importance [7, 8]. The scope of CAD application is constantly expanding, covering not only traditional tasks of functional-logical and electrical modeling but also such complex stages as optimization of placement of irregularly shaped elements and elements with different dimensions [9, 10].

Moreover, the existing methods for automated very-large-scale integration (VLSI) cell layout design exhibit a number of significant limitations that hinder their effective utilization under modern conditions [11-13]. An analysis of traditional algorithms, such as the Gordian method, the Gamma algorithm, and the minimum graph cut algorithm, reveals their lack of flexibility when dealing with non-standard geometric components and complex topologies of interconnect spaces [5, 14]. It should be noted that most classical approaches assume of a simple shape of the placed elements and the homogeneity of the routing space, which does not correspond to the current realities of highly integrated circuit design [15].

The process of automating the design of element placement can be divided into two key tasks: optimizing the mutual arrangement of components of various shapes and sizes, and tracing connections between them while considering the complex geometry of the routing field [16, 17]. Solving these tasks requires the development of new models and methods capable of accounting for the diversity of component shapes and sizes, as well as the complex geometry of modern electronic devices [18].

The objective of this work is to develop and analyze novel methods and programs for optimizing the placement of irregularly shaped elements in a complex-shaped routing field, based on a combinatorial analogue of the coordinate descent method considering geometric constraints, the method of weight and penalty coefficients, and a multistart procedure. The proposed approach aims to provide the capability for efficient handling of non-standard component configurations and intricate routing field topologies [3].

The stated goal is achieved by solving several interrelated tasks. First, a mathematical model is generalized by developing the classical quadratic assignment problem with extended topological variants. Second, an algorithmic framework is developed concerning the combinatorial placement optimization method. Third, the proposed models and algorithms are realized programmatically in high-level languages. Fourth, computational experiments are performed on test problems containing non-standard elements and complex configuration routing fields. Finding test problems with known solutions is a non-trivial task even for the classical variant of the quadratic assignment problem. In this regard, an original "snake" type test has been developed, characterized by convenience and practicality in application to a very wide range of object placement problems.

The scientific novelty of the work is in topologically widening possibilities when solving the problem of optimal placement, the approach allows using components of any shape – non-standard geometry, as well as restrictions zones or deformations of the routing field for both planar and three-dimensional problems. This significantly extends applications compared to classical algorithms and opens new prospects for optimization topology of modern integrated circuits [19].

The practical significance of the research is in being able to apply the developed models and methods to real problems in the design of electronic devices, considering that requirements for design and specifications are of these days [20]. The work's findings can be applicable to design highly integrated VLSI what is a multilayer printed board circuit boards and other complex electronic systems [21].

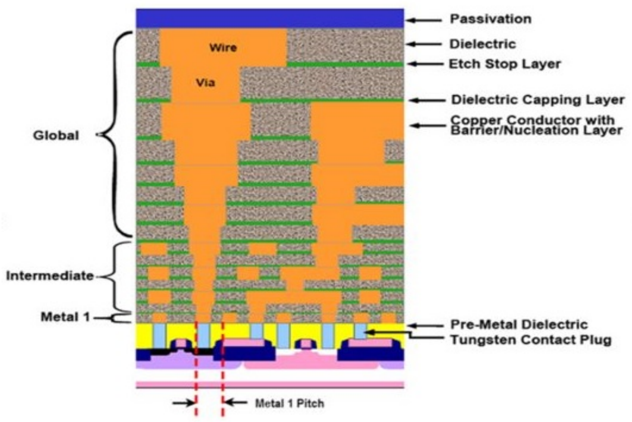


Fig. 1. Cross-sectional view of VLSI topology

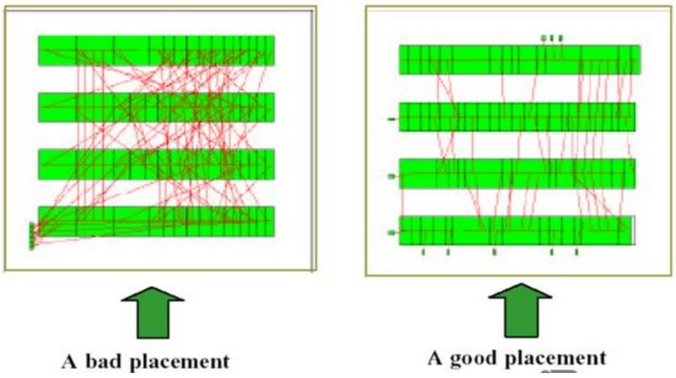


Fig. 2. The placement problem

Tasks of spatial planning design in production

Layout or spatial planning solution for production is an operation of constructional production design, which results in determining the composition of production premises, their dimensions and rational mutual arrangement, as well as executing drawings of floor plans and sections at a certain scale.

This involves solving the following tasks: selecting the type of building structure; determining the composition of production premises, their sizes and rational relative positioning; placing equipment inside the premises; routing intra-shop pipelines; choosing and locating pipeline fittings; and defining optimal parameters of transport and pipeline networks (Fig. 3, 4) [22].

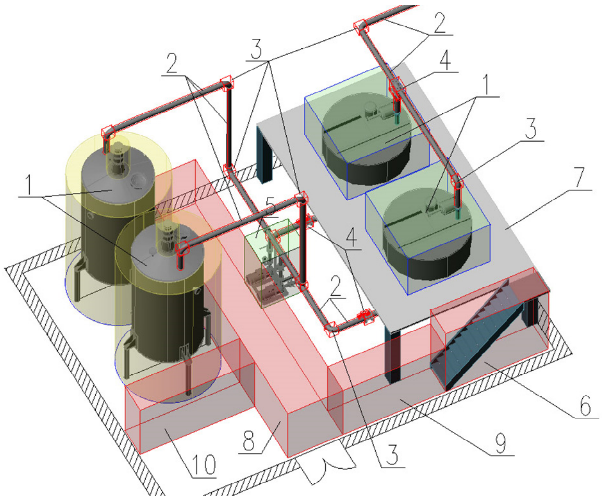


Fig. 3. Description of objects in an equipment layout fragment: 1 – vessels; 2 – pipelines; 3 – pipeline fittings; 4 – pipeline valves; 5 – pump block; 6 – ladder; 7 – service platform; 8, 9 – passages; 10 – vessel service area

Various criteria and constraints are applied when solving this problem [7, 9, 16, 20, 23]. The calculation results provide the coordinates for the placement of individual equipment elements on the column grid (CG) – an analogue of the routing field in electronics, and the topological characteristics of their interconnections.

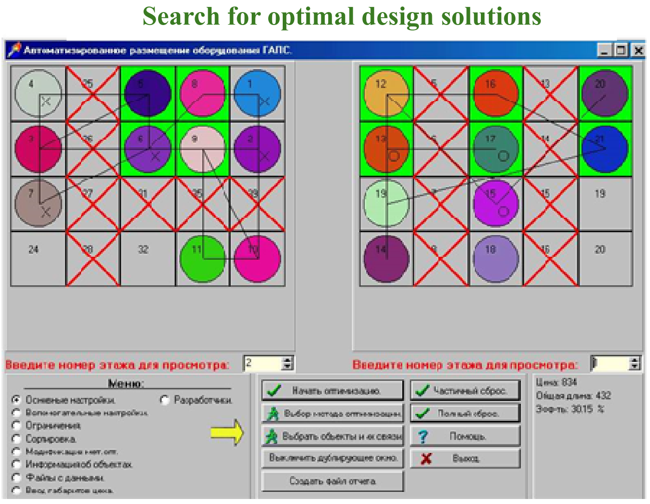


Fig. 4. 5*4 column grid (MCVD) of interconnections

Optimization problem formulation

The interconnection field (IF) is formed using a rectangular grid and, in the classical two-dimensional case, is described by the following parameters: n_x, n_y, h_x, h_y, where n_x is the number of positions in a horizontal row; n_y is the number of positions in a vertical row; h_x is the horizontal step between positions; h_y is the vertical step between positions. Let us assume that only the simplest form elements (occupying one cell of the IF) $e_1,\ldots,e_n$ are given, and for each pair of them, weights $r_{ij}(i,j=1,\ldots,n)$ are specified, defining the "degree of connection" of these elements and forming the connection matrix $R=\{r_{ij}\}_{i,j=1,\ldots,n}$.

There is a set of positions for placing elements $p_1, p_2, \ldots, p_m$ ($m \geq n$). Without loss of generality, we will assume that $m=n$. If $m>n$, then $m-n$ fictitious elements are introduced that have no connections with any elements of the interconnection field.

Elements of irregular shape are formed from the simplest ones by assigning a sufficiently large "degree of connection". This method of modeling elements with complex topology is analogous to the synthesis of high-molecular compounds with strong chemical bonds. As is well known, due to the significant strength of interatomic bonds, these compounds are characterized by their stability. In placement problems, a high "degree of connection" guarantees the formation of elements with a given topology. There exist rational values of the "degree of connection" magnitude, which can be easily determined using computational experiments.

Let us define the distances $d_{ij}(i,j=1,\ldots,n)$ between arbitrary pairs of positions, which form a symmetric matrix $D=\{d_{ij}\}_{i,j=1,\ldots,n}$ with a zero main diagonal $d_{ii}=0$ $(i=1,\ldots,n)$. Various metrics can be used to calculate the elements of matrix D, among which the most employed are the orthogonal and Euclidean metrics [3], [6], [9], [11].

The placement of basic elements in arbitrary positions can be represented as a permutation $p=(p_1,\ldots,p_n)$, where p_i denotes the position number assigned to the i-th element. The total number of distinct placement configurations is n!, a significantly increasing factor in the computational complexity of the global optimization problem as the number of elements grows.

A fragment of the switching field, as illustrated in Figure 5, is considered. Specifically, the length of connections between elements e_i and e_j is assessed using the magnitude of:

$$L_{ij} = r_{ij} d_{p(i)p(j)} \quad (i, j = 1,...,n) \tag{1}$$

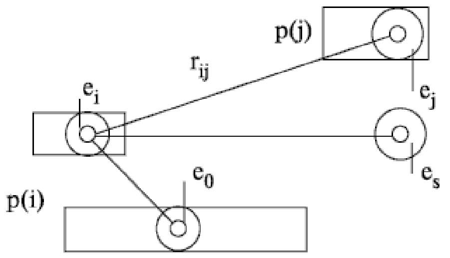


Fig. 5. Representation of the switching field

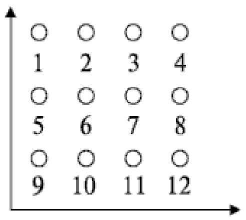


Fig. 6. Example of a rectangular switching field: a fixed set of 12 positions

We define E_S as the set of all elements with fixed positions, including element e_0. Specifically, the total weighted length of connections between element e_i and elements from E_S is assessed using the following formula:

$$a_{ip(i)} = \sum_{s \in E_s} r_{is} d_{p(i)s} \quad (i = 1,...,n) \tag{2}$$

where $d_{p(i)s}$ - specifically denotes the distance between element e_i situated in position p_i and the element in question.

Optimization for placing elements on a switching field begins. The development of a mathematical model incorporating the geometric parameters of the elements as well as the characteristics of the switching field. Typically, the optimization criterion is the minimization of the weighted length (MWL) of connections. Various other optimization problem variants exist, including those based on complex criteria and multi-criteria problems.

Given the symmetry of matrices R and D, the expression for minimizing the total weighted length of connections for arbitrary placement can be formulated as:

$$F(p) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij} d_{p(i)p(j)} + \sum_{i=1}^{n} a_{ip(i)} \to \min_{p \in \Omega} \tag{3}$$

where Ω denotes the set of permissible permutations.

The placement problem for the orthogonal metric specifically entails minimizing the functional according to the MWL criterion of connections:

$$F(p) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij} \left(|x_i - x_j| + |y_i - y_j| \right) + \\ + \sum_{i=1}^{n} \sum_{s \in E_s} r_{is} \left(|x_i - x_s^0| + |y_i - y_s^0| \right) \to \min_{(x,y) \in \Xi}. \tag{4}$$

where Ξ denotes the set of coordinates representing permissible element placements on the switching field.

Notably, this problem constitutes a variant of the general mathematical model known as the binary distribution problem [8, 9, 13, 18]. The geometric constraint stipulates that no more than one element can occupy a single cell. Furthermore, the complex topology of the switching field can be accounted for by establishing forbidden zones within a specified rectangular switching field, utilizing the established penalty function method. These penalties are subsequently incorporated into the primary objective function:

$$F^*(p) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij} d_{p(i)p(j)} + \sum_{i=1}^{n} a_{ip(i)} + \\ + F_{penalty}(p) \to \min_{p \in \Omega}. \tag{5}$$

The switching field and elements exhibiting three-dimensional topology are characterized analogously through the incorporation of a third Cartesian variable, z [24].

Development of an optimization algorithm for the mutual placement of elements of various sizes

In this section, we put forward an approach to solving the problem of optimal location of items of different shapes and sizes on a multidimensional switching field with a complicated topology. The developed method is based on a combinatorial version of Gauss-Seidel algorithm, modified for non-uniform elements [25].

Coordinate descent method description

We address the integer optimization problem defined by an objective function $F(x)$, where x is a vector of parameters to be optimized, specifically a permutation of position numbers for n elements without repetitions. Such a computation of cell coordinates for item placement does implicitly consider geometrical constraints that go with the position number. Given the algorithmic complexity of exiting the penalty region, traditional optimization methods for continuous functions are unsuitable. Consequently, the focus must be on exploring the space of permissible permutations without repetitions.

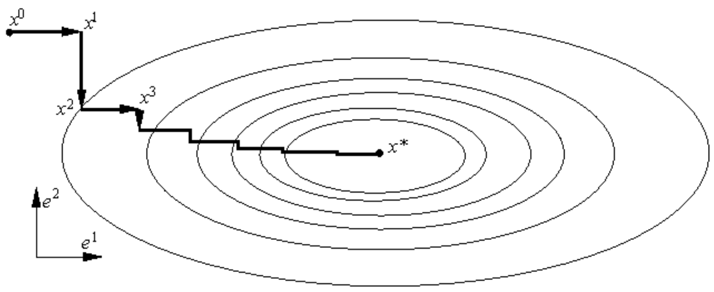


Fig. 7. Classical method of coordinate descent for two independent variables

After executing a trial step along a coordinate, the permutation is adjusted: the argument whose value coincides with the new value of the varied coordinate is identified. This argument's value is then restored to its original state, as it was before the optimization step, of the varied coordinate.

This process ensures a return to the space of permutations without repetitions. In this combinatorial variant of the coordinate descent method, two coordinates are modified simultaneously in one computational step, unlike the single-coordinate modification in the conventional method. A standard trial step is executed along one coordinate, while the other is corrected to ensure a return to the permissible region. If an improvement is achieved, the identified point serves as the new starting point. Otherwise, a step is executed along another coordinate with concurrent adjustment of the vector of element position numbers, ensuring a return to the permissible region.

The modeling of complex-shaped elements is realized through their composition from elementary cells within the switching field. This process is facilitated by the strategic assignment of weight coefficients to the connections between the involved elementary cells. Notably, these coefficients exhibit a penalty character, inducing a sharp increase in the objective function when the corresponding components of the complex element configuration are mutually separated.

The judicious selection of weight and penalty coefficients is of paramount importance for the performance of the optimization method. When the values of these coefficients are underestimated, it leads to the blurring of the calculated configurations of complex elements and the boundaries of the switching field. On the other hand, overestimating the values of these coefficients does not contribute to an acceleration of the algorithm.

The algorithm we propose follows an iterative approach where, at each iteration, a local optimization of the positions of individual components is performed. That is, for each component, the allowable set of locations is determined, considering both the component dimensional characteristics and the free-space configuration within the switching field.

The algorithm we propose is an iterative approach where in each iteration a local optimization of the positions of individual components is done. That is for each component, it determines the allowable set of locations considering both the dimensional characteristics of the component as well as free space configuration within the switching field.

Our algorithm integrates a multistart approach; this means that stochastic initial placements are generated intermittently and then followed by a specialized optimization for each case. This can make the attainment of the global optimum much more likely.

The proposed algorithm was shown to be highly effective in finding near-optimal solutions for a wide range of topological problems typical in the design of modern integrated circuits, electrical networks, and equipment configuration.

Algorithmic implementation considerations

The system realized is developed using the PascalABC programming language and integrates modules for optimal placement of elements along with complete visualization and tools for analysis of results.

The architecture of the software system is constructed based on a modular paradigm, facilitating system flexibility and extensibility. The primary modules encompass:

1. An input and preprocessing subsystem that enables the specification of element parameters and switching field characteristics.
2. A placement optimization module implementing the developed combinatorial algorithm.
3. A comprehensive visualization and results analysis subsystem.

A salient characteristic of the developed software system is its flexibility and capability to handle elements of arbitrary geometries on switching fields exhibiting complex topological structures. The integrated system facilitates improved efficiency in the development process of contemporary VLSI circuits.

Computational experiments and analysis of results

A complete analysis was carried out to confirm the effectiveness of the proposed methodology based on several computational experiments. The approach could possibly be extended to other fields that involve complex geometries and physical phenomena, like calculating three-dimensional magnetic fields in systems with both permanent magnets and ferromagnetic materials [26].

Special test cases were non-typical and very complicated switching field configurations. The development of test cases for placement problems poses a significant challenge, even in the case of the classical quadratic assignment problem [27]. Moreover, when the problem is further complicated by the introduction of topological factors, the difficulty of testing increases considerably.

The authors successfully addressed the critical challenge of testing through the ingenious development of an original multi-dimensional "snake-like" test characterized by a known exact solution.

Example 1. The optimization of element placement on a switching field.

A representative example is the problem of optimizing the placement of 36 elements on a 6x6 switching field subject to non-trivial geometric constraints. The specifics of this problem are as follows: the switching field has dimensions of 6x6 cells, with a forbidden zone of size 3x3 defined in the lower left corner. The element composition includes 18 standard square elements of unit size 1x1 (identified by numbers 4 through 21), two non-standard (complex) elements of square shape with dimensions 2x2 (formed by cells of simplest elements numbered 22, 23, 24, 25) and triangular shape (formed by cells of simplest elements numbered 1, 2, 3). Additionally, 11 fictitious elements (numbers 26 through 36) are introduced to account for the presence of empty (unoccupied by elements) space on the switching field. The initial approximate placement and the result of global placement optimization are presented in Figures 8 (A) and (B). The calculations were performed using Manhattan distance. In the calculation, the connection coefficients of simple elements were assumed to be equal to 1. The values of the connection coefficients for complex elements were set equal to 100. The value of the penalty function for the forbidden area was 2000. It is noteworthy that these values of weight and penalty coefficients were consistently present in all subsequent examples.

The results of the optimization, shown in Figure 8, demonstrate how well the developed algorithm works for very complex placement problems. Remarkably, the computational runtime for the complete optimization process was only 5 minutes.

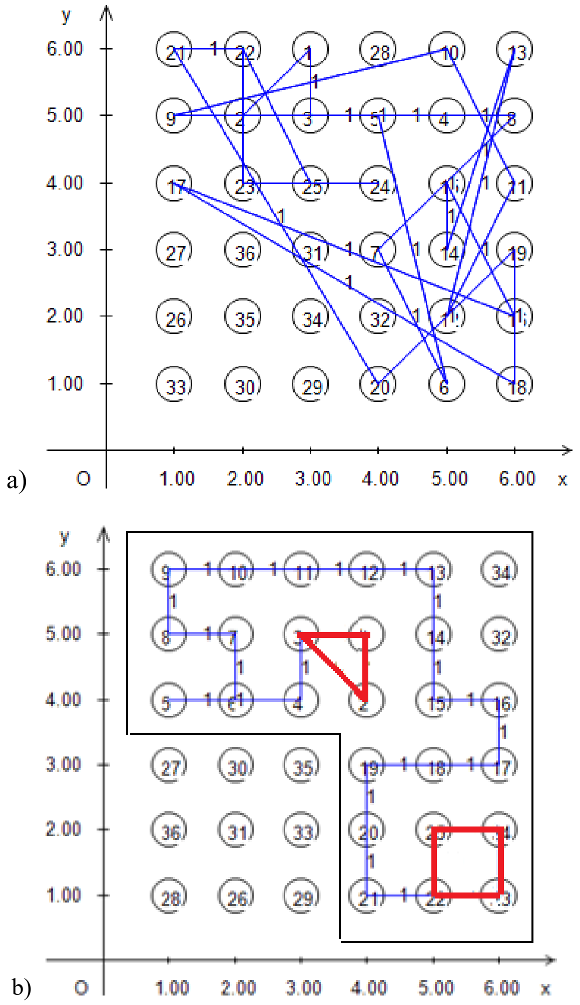


Fig. 8. The initial approximation (a) and the globally optimal element placement (b) computed by the program on a switching field characterized by the presence of non-standard elements and a forbidden zone, utilizing the Manhattan distance metric

Example 2. Optimization of placements based on Euclidean distance.

The forbidden region is defined by the following inequalities:

$$2 \le x \le 5, 3 \le y \le 4. \quad (6)$$

The switching field under consideration consists of a total of 36 cells, of which 18 cells are occupied by real elements and the remaining 18 by fictitious elements (empty cells). The element composition includes 11 standard square elements of unit size 1x1 (identified by numbers 4 through 14), two non-standard (complex) elements of square shape with dimensions 2x2 (formed by cells of simplest elements numbered 15, 16, 17, 18) and triangular shape (formed by cells of simplest elements numbered 1, 2, 3). The optimization process involves a total of 1010 iterations. Notably, every 5 iterations, in the absence of improvement in the objective function value by the Gauss-Seidel method, a new start is carried out, employing a uniform random distribution of elements in the feasible region while excluding the forbidden zone.

The initial approximate placement was determined using 100 iterations of the Monte Carlo method, uniformly distributed over the switching field. The results of the subsequent placement optimization process are shown in Figures 9 and 10. An important point is that the forbidden region, which fills its major function in bounding the placement solution space, lies inside the switching field marked out by a contour in the shape of a rectangle.

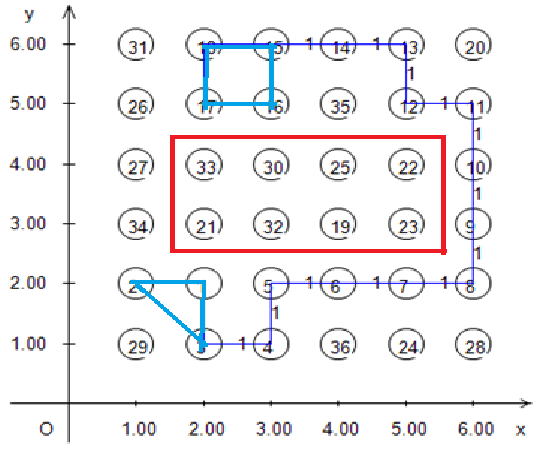


Fig. 9. The program-computed optimal placement of elements on a switching field characterized by the presence of non-standard elements and a forbidden zone, utilizing the Euclidean distance metric

```
k = 101  fmin = 847.22  fmin0 = 755.84  i = 1  fm1 = 755.84  fm2 = 755.84
k = 202  fmin = 760.35  fmin0 = 754.25  i = 2  fm1 = 754.25  fm2 = 754.25
k = 303  fmin = 758.84  fmin0 = 753.84  i = 3  fm1 = 753.84  fm2 = 753.84
k = 404  fmin = 758.27  fmin0 = 753.84  i = 4  fm1 = 753.84  fm2 = 753.84
k = 606  fmin = 886.86  fmin0 = 753.84  i = 1  fm1 = 753.84  fm2 = 753.84
k = 707  fmin = 841.49  fmin0 = 753.84  i = 2  fm1 = 753.84  fm2 = 753.84
k = 808  fmin = 842.98  fmin0 = 753.84  i = 3  fm1 = 753.84  fm2 = 753.84
k = 909  fmin = 757.07  fmin0 = 753.42  i = 4  fm1 = 753.42  fm2 = 753.42

Result:     Shtraf =0  fopt =753.42135623731
```

Fig. 10. The results of iterative optimization calculations performed using the Euclidean distance metric

Figure 10 gives the results of iterative optimization computations. Here, k stands for the iteration number, fmin is the value of the objective function at the k-th iteration, and fmin0 is the best value of the objective function from the 1st to the k-th iteration. Moreover, fm1 and fm2 represent in which case they coincide a new start being. If these values coincide, a new start is initiated to avoid premature convergence and ensure a thorough exploration of the solution space. Shtraf represents the value of the penalty function for the obtained solution and serves as a criterion of constraint satisfaction; fopt corresponds to the calculated value of the global optimum.

Remarkably, the total computation time took only 5 minutes, showing what an efficient optimization algorithm, it was.

Example 3. The optimization of placements utilizing the Euclidean distance. metric, with explicit consideration for the presence of terminals and a forbidden region on the switching field.

The forbidden region is described by the following inequality:

$$\left(x-3.5\right)^2 + \left(y-3.5\right)^2 \le 4 \quad (7)$$

In this optimization example, the element composition and number of iterations remain consistent with those in Example 2. The initial placement and the results of the placement optimization are illustrated in Figures 11 and 12. A circular forbidden region, delineated by a circumference, is situated at the center of the switching field. The terminals (inputs and outputs) of the circuit are positioned at points with coordinates (0.1, 5) and (0.1, 2), and they must be directly connected to elements number 1 and 18, respectively. These elements are integral components of the complex elements denoted by Roman numerals I and II in Figure 11b.

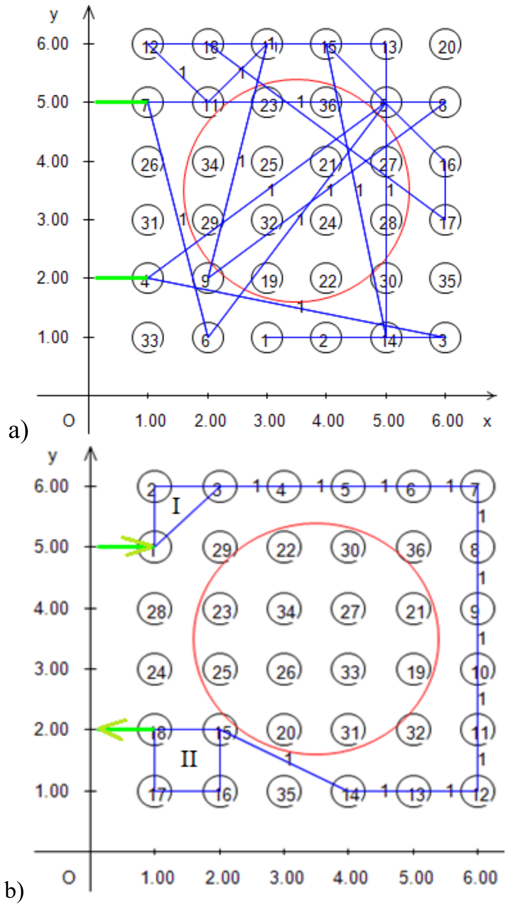


Fig. 11. The initial approximation (a) and the globally optimal placement of elements (b) computed by the program on a switching field with non-standard elements, terminals, and a circular forbidden zone, utilizing the Euclidean distance metric

```
k = 101   fmin = 1294.68   fmin0 = 847.41   i = 1   fm1 = 847.41   fm2 = 847.41
k = 202   fmin = 844.66   fmin0 = 844.66   i = 2   fm1 = 844.66   fm2 = 844.66
k = 303   fmin = 1242.14   fmin0 = 844.66   i = 3   fm1 = 844.66   fm2 = 844.66
k = 404   fmin = 1029.81   fmin0 = 844.66   i = 4   fm1 = 844.66   fm2 = 844.66
… … … … … … … … … … … … … … … … … … … …
k = 1818   fmin = 999.22   fmin0 = 844.66   i = 3   fm1 = 844.66   fm2 = 844.66
k = 1919   fmin = 1223.76   fmin0 = 844.66   i = 4   fm1 = 844.66   fm2 = 844.66

Result:     Shtraf =0   fopt =844.657424214809
```

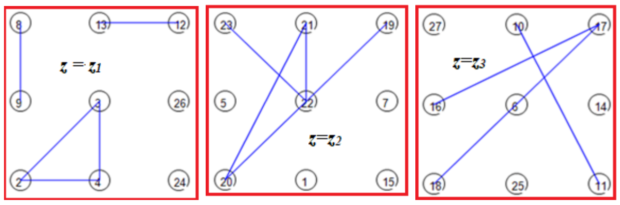
Fig. 12. The results of iterative optimization calculations performed using the Euclidean distance metric

The total computation time amounted to a mere 5 minutes.

Example 4. Optimization of placements for elements with diverse geometries on a three-dimensional switching field utilizing the euclidean distance metric.

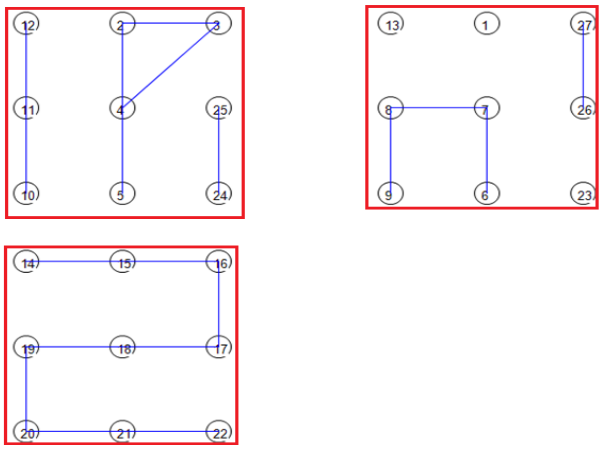
The switching field is modeled as a 3x3x3 cube, consisting of 27 elementary cells. The element composition includes 23 standard square elements of unit size 1x1, identified by numbers 5 to 27, and a non-standard (complex) tetrahedral element formed by combining the cells of the simplest elements with numbers 1, 2, 3, and 4. The results of the placement optimization process are presented in Figures 13 and 14, which illustrate cross-sections of the cubic switching field in various planes, providing a comprehensive view of the optimized element arrangement.

Figures 13 and 14 illustrate the results of the placement optimization process, depicting cross-sections of the cubic switching field in multiple planes.



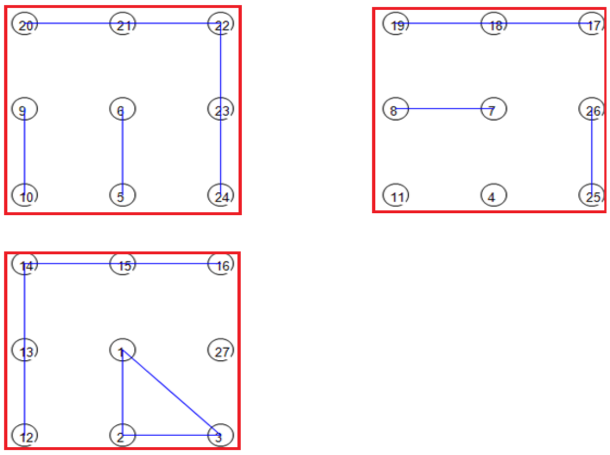
a)

Cross-sections in the *xOy* plane; $z = z1, z2, z3$



b)

Cross-sections in the *xOz* plane; $y = y1, y2, y3$



c)

Fig. 13. A comparative analysis of cross-sections of the three-dimensional switching field. The initial approximation (a) and the computed globally optimal placement of elements are illustrated in the *xOy* plane (b) and in the *xOz* plane (c), highlighting the application of the Euclidean distance metric on a switching field featuring non-standard elements

k = 4949 fmin = 749.33 fmin0 = 747.26 i = 4 fm1 = 747.26 fm2 = 747.26
fm2 = 747.68
Result: fopt =747.26 fm2 = 747.68

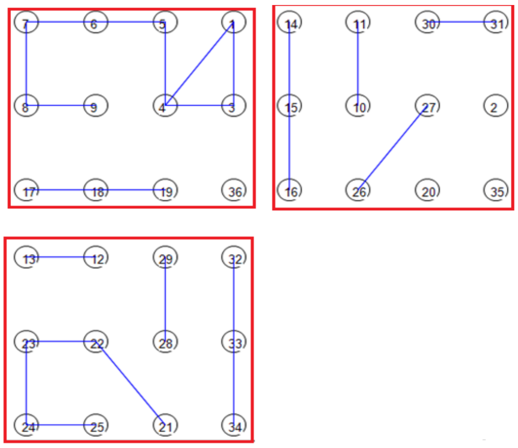
Fig. 14. The results of iterative optimization calculations performed using the Euclidean distance metric

The total computation time amounted to a mere 5 minutes.

Example 5. The optimization of placements for elements with diverse geometries on a three-dimensional switching field, utilizing the Euclidean distance metric as the objective function.

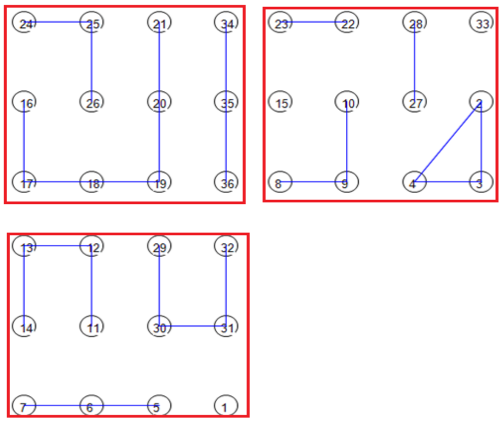
The switching field is modeled as a parallelepiped with dimensions 4x3x3, consisting of 36 elementary cells. The elemental composition consists of thirty-two standard square elements of unit size 1x1, labelled by numbers 5 to 36, and one non-standard (compound) tetrahedral element formed by the union of the cells of the basic elements with numbers 1, 2, 3, and 4. The results of the placement optimization process are presented in Figures 15 and 16, which illustrate cross-sections of the parallelepiped switching field in various planes parallel to the axes of the Cartesian coordinate system. It is noteworthy that this problem differs from Example 4 in its larger dimensionality, which exponentially increases the computational complexity of the optimization, thereby underscoring the need for efficient algorithms in handling such complex scenarios.

Cross-sections in the *xOy* plane; $z = z1, z2, z3$



a)

Cross-sections in the *xOz* plane; $y = y1, y2, y3$



b)

Fig. 15. Cross-sections of the three-dimensional switching field and the computed globally optimal placement of elements: (a) - cross-sections parallel to the *xOy* plane, (b) - cross-sections parallel to the *xOz* plane (variant with Euclidean distance metric)

k = 15049 fmin = 760.10 fmin0 = 757.09 i = 4 fm1 = 757.09 fm2 = 757.09
Result: fopt =757.09 fm2 = 757.09

Fig. 16. The results of iterative optimization calculations performed using the Euclidean distance metric

The total computation time amounted to a mere 10 minutes.

Conclusion

One new way to optimize element location on the switch field has been developed and probed, which may in practice bring several significant benefits over traditional approaches. The proposed combinatorial analog of the coordinate descent method exhibits high efficiency in solving discrete-variable placement problems. The developed algorithm has been successfully adapted for the optimization of element placements and fields with non-standard geometric configurations. A novel multidimensional snake-type test has been devised to effectively evaluate this method.

In optimization algorithms, the multistart procedure with local optimization methods greatly improves the chances of finding the global optimum. This feature is particularly important for very high-dimensional problems with a complicated landscape of the objective function.

Extensive experimental studies on a wide range of test problems have convincingly confirmed the high adaptability of the method to various variations in placement conditions. The algorithm demonstrated the ability to function effectively with both standard elements and non-standard shaped components using different metrics.

In applied realms, the potential use of this method in designing modern electronic and other devices based on findings is of prime importance [28]. Specifically, the ability of the approach to deal with sophisticated geometric constraints and work productively with non-uniform fields makes it widely possible to improve the design of complicated systems that have non-standard configurations.

Further research in this direction may focus on tailoring the approach to various specific needs of different application areas or on developing parallel computing strategies. These developments would help improve algorithmic performance in solving ultralarge problems, which is increasingly important for high-tech applications today.

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КОМБИНАТОРНЫЙ ПОДХОД К ОПТИМИЗАЦИИ РАЗМЕЩЕНИЯ ЭЛЕМЕНТОВ НЕРЕГУЛЯРНОЙ ФОРМЫ НА ДВУМЕРНЫХ И ТРЕХМЕРНЫХ КОММУТАЦИОННЫХ ПОЛЯХ СЛОЖНОЙ ТОПОЛОГИИ

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Аннотация
В статье представлен новый метод оптимизации размещения элементов нестандартной конфигурации на двумерных и трехмерных коммутационных полях со сложной топологией. Разработан комбинаторный аналог метода Гаусса-Зейделя, адаптированный для эффективной работы с компонентами нерегулярной формы и различных габаритов. Алгоритм учитывает специфические особенности коммутационного поля, включая наличие запретных зон и неоднородность структуры. Для повышения вероятности нахождения глобального оптимума применена процедура мультистарта. Создана программная реализация с модулями оптимизации размещения и визуализации результатов. Эффективность подхода проверена на оригинальных тестовых задачах типа "многомерной змейки". Рассмотрены примеры для двумерных и трехмерных полей с элементами различных размеров и геометрическими ограничениями. Результаты демонстрируют адаптивность метода к разным условиям размещения элементов нестандартной формы. Достигнуто квазиоптимальное размещение с погрешностью 0,055% за 1010 итераций при времени счета 5 минут для двумерных и 10 минут для трехмерных задач. Метод применим при проектировании интегральных схем, компоновке электротехнических устройств и оптимизации размещения оборудования в производственных системах. Представлены количественные оценки эффективности алгоритма для полей различной сложности. Проанализированы особенности реализации для компонентов с нетривиальной геометрией и коммутационных полей нестандартной топологии. Выявлены преимущества предложенного подхода по сравнению с классическими методами оптимизации размещения разногабаритных элементов. Определены направления дальнейших исследований, включая адаптацию к специфическим требованиям различных областей применения и разработку стратегий параллельных вычислений для задач большой размерности с элементами нерегулярной формы на многомерных коммутационных полях сложной топологии.

Ключевые слова: *оптимизация размещения, элементы нерегулярной формы, многомерные коммутационные поля, комбинаторный метод Гаусса.*

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