THE METHOD OF MOMENTS IN THE PROBLEM OF ESTIMATING THE PARAMETERS OF A COMMUNICATION CHANNEL

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This paper introduces a synthesis of an algorithm for estimating the parameters of a communication channel based on the observed M-QAM signal with a known information sequence. The proposed algorithm leverages the method of moments, expressed in the form of the Tikhonov A.N. functional, which allows for the efficient estimation of various channel parameters. These parameters include the amplitude, phase, and frequency shift of the received signal. The phase noise is also incorporated into the phase model to provide a more accurate reflection of real-world channel conditions. The estimation process was carried out under conditions of additive white Gaussian noise (AWGN), as well as in scenarios where the noise followed a lognormal probability distribution. The phase was assumed to follow a uniform distribution. To evaluate the performance of the proposed algorithm, a computational experiment was conducted, focusing on the accuracy of parameter estimation for a 64-QAM signal. The results obtained using this method were compared with those achieved through advanced Kalman filtering, a well-known approach for parameter estimation in communication systems. Additionally, a rough analysis of the computational complexity of the algorithms was performed, comparing the method of moments to recursive nonlinear filtering algorithms. Experimental curves depicting the interference immunity of 64-QAM signal reception were obtained using both the synthesized algorithm and the advanced Kalman filtering method. These results provide valuable insights into the effectiveness and practical feasibility of the proposed approach for parameter estimation in communication channels, particularly in noisy environments. To further validate the robustness of the proposed algorithm, different noise levels and channel conditions were tested in the simulations. This allowed for a comprehensive assessment of the algorithm's adaptability and accuracy under varying signal-to-noise ratios (SNR), ensuring its applicability across a wide range of practical communication scenarios.

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Introduction

The problem of the estimation signal parameters is actual because it allows to implement quasicoherent signal reception the interference-immunity which is higher than a noncoherent reception. The accuracy of the estimation effects on an error probability of an information symbol reception. Especially, it occurs in the case of a multiposition signals reception such as 64, 256, 1024-QAM which are used for increasing the transmission speed. The problem of a definition of unknown communication channel parameters with AWGN is well researched especially in systems with one receive and one transmission antennas (SISO). A more complicated case is a reception with nongaussian interfering noise [1-4]. An additive signal processing is often used [5-7] or are applied algorithms which are effective in cases prior uncertainty in respect of laws of noise distribution [8-11]. In the first case a computational complexity of procedures may be high, in the second case a low accuracy of unknown parameters can be obtained due to an absence of priory data [8-10].

This paper proposes the signal reception with interference noise with lognormal amplitude and uniform phase distribution which is dominating over AWGN. For example, industrial and atmospheric interference belong to this interference effect [12-14, 21-26]. The density of probability distribution of this interference noise is described by a complex integral expression. The additive mixture of AWGN and interference effect is more complicated and includes integral expression which does not have analytical solution. This makes it difficult to use optimal approaches, such as a maximum likelihood method or minimum mean squared error (MSE) for estimation of the signal parameters of communication channel. To simplify the algorithm was used prior information of expectation value and its median. It allows to use the method of moments for solving the problem. This trade-off between computational efficiency and estimation accuracy is a critical consideration in the design of robust communication systems.

In more advanced systems, such as multiple-input multiple-output (MIMO) configurations, where multiple antennas are used for transmission and reception, the estimation problem becomes even more challenging due to the increased number of parameters and the potential for more complex noise and interference environments. Therefore, the development of efficient algorithms capable of handling nongaussian noise with low computational complexity remains a key research area in modern communication systems.

Formulation of the problem. In this article are observed quadratures y_{ci}, y_{si} M-position signal quadrature amplitude modulation (M-QAM) with the mixture of AWGN and an interference noise with a lognoramal amplitude distribution and a uniform phase distribution:

$$\mathbf{Y}_i = \mathbf{\Phi}_i(\mathbf{\Theta}_i) + \boldsymbol{\gamma}_i \ . \qquad (1)$$

where $\mathbf{Y}_i = \begin{pmatrix} y_{ci} & y_{si} \end{pmatrix}^T$, $\mathbf{\Theta}_i = \begin{pmatrix} A_i & \Delta f_i & \varphi_i \end{pmatrix}^T$ – vector of unknown signal parameters, components of which are amplitude A_i, frequency shift Δf_i and phase φ_i of the signal correspondingly,

$$\mathbf{\Phi}_i(\mathbf{\Theta}_i) = \begin{pmatrix} h_{ci}(\mathbf{\Theta}_i) I_i - h_{si}(\mathbf{\Theta}_i) J_i & h_{si}(\mathbf{\Theta}_i) I_i + h_{ci}(\mathbf{\Theta}_i) J_i \end{pmatrix}^T$$

– vector-function which is described quadrature components of the M-QAM signal, $h_{ci}(\mathbf{\Theta}_i) = A_i \cos(2\pi \Delta f_i T_c i + \varphi_i)$, $h_{si}(\mathbf{\Theta}_i) = A_i \sin(2\pi \Delta f_i T_c i + \varphi_i)$ – multipliers of the communication channel which are changing over time and dependent on the unknown vector of parameters $\mathbf{\Theta}_i$, T_c – duration information and test symbol I_i, J_i, $\boldsymbol{\gamma}_i = \begin{pmatrix} \gamma_{ci} & \gamma_{si} \end{pmatrix}^T$ is interference effect's vector of quadratures, $\gamma_{ci} = \eta_{ci} + \mu_{ci}$; $\gamma_{si} = \eta_{si} + \mu_{si}$, μ_{ci}, μ_{si} – components of AWGN with zero mathematical expectation and variance $E(\mu_{ci}^2) = E(\mu_{si}^2) = \sigma_\mu^2$, η_{ci}, η_{si} – quadrature components of the interference effect, $E(\eta_{ci}) = E(\eta_{si}) = 0$, $E(\eta_{ci}^2) = E(\eta_{si}^2) = \sigma_\eta^2 = \frac{1}{2} e^{2(\nu + \sigma^2)}$, ν, σ^2 are lognormal function's parameters of the probability density function, $E(\gamma_{ci}) = E(\gamma_{si}) = 0$; $E(\gamma_{ci}^2) = E(\gamma_{si}^2) = \sigma_\gamma^2 = \sigma_\eta^2 + \sigma_\mu^2$, $E(\cdot)$ is expectation operator, «T» is transposition operator.

Required by the testing sequence in the form of M-QAM signal by observing the signal $\mathbf{Y}_i$, $i = 1,...,m$, obtain an estimation of the channel multipliers $h_{ci}(\widehat{\mathbf{\Theta}}_i)$, $h_{si}(\widehat{\mathbf{\Theta}}_i)$.

Solving the problem. Obtaining an estimation parameter of a signal is related to search an estimation the vector $\mathbf{\Theta}_i = \begin{pmatrix} A & \Delta f & \varphi_i \end{pmatrix}^T$ of unknown signal parameters. It proceeding using known moments of the interference signal [15]. In this paper the sample mean and median is used as a moment. The synthesizing of the communication channel multipliers $h_{ci}(\mathbf{\Theta}_i)$, $h_{si}(\mathbf{\Theta}_i)$ estimation algorithm is carried out under the condition of strong dominance of the interference signal over the Gaussian noise or in its absence.

Using the model (1) form a random value a_i which is the amplitude of the interference signal with the lognormal distribution and the mathematical expectation $E(a_i) = e^{\nu + \frac{\sigma^2}{2}}$:

$$a_i = \left\| \mathbf{Y}_i - \mathbf{\Phi}_i(\mathbf{\Theta}_i) \right\| =$$

$$= \sqrt{(y_{ci} - h_{ci}(\mathbf{\Theta}_i) I_i + h_{si}(\mathbf{\Theta}_i) J_i)^2 + (y_{si} - h_{si}(\mathbf{\Theta}_i) I_i - h_{ci}(\mathbf{\Theta}_i) J_i)^2} = S_i(\mathbf{\Theta}_i)$$

where $h_{ci}(\mathbf{\Theta}_i) = A_i \cos(2\pi \Delta f_i T_c i + \varphi_i)$, $h_{si}(\mathbf{\Theta}_i) = A_i \sin(2\pi \Delta f_i T_c i + \varphi_i)$, T_c – test and information symbols I_i, J_i duration of the M-QAM signal, i is discrete time. We find the sample average of a and equate it to mathematical expectation $E(a_i)$. The result model:

$$e^{\nu + \frac{\sigma^2}{2}} \approx \frac{1}{m} \sum_{i=1}^{m} S_i(\mathbf{\Theta}_i) \cdot \qquad (2)$$

Next, we recursively solve the nonlinear equation (2) of the relative unknown vector $\mathbf{\Theta}$ using the theory of recursive nonlinear Stratanovich filtering. We suppose that the amplitude and the frequency shift are slowly changing during time processes, and $\varphi_i = \varphi + \zeta_i$, where ζ_i is phase noise. Then, only the invariable component φ will be estimated. Accounting (2), we write model

$$\mathbf{\Theta}_l = \mathbf{\Theta}_{l-1} + \boldsymbol{\xi}_l, \; m e^{\nu + \frac{\sigma^2}{2}} = S(\mathbf{\Theta}_l) + \varepsilon_l, \qquad (3)$$

where $S(\mathbf{\Theta}_l) = \sum_{i=1}^{m} S_i(\mathbf{\Theta}_l)$, $\boldsymbol{\xi}_l$ is noise of dynamic system with $E(\boldsymbol{\xi}_l) = \mathbf{0}_{3\times1}$, $E(\boldsymbol{\xi}_l \boldsymbol{\xi}_l^T) = \sigma_\xi^2 \mathbf{I}_{3\times3}$, $\sigma_\xi^2 \to 0$, $\mathbf{I}_{3\times3}$ is unity matrix with size 3×3, ε_l is error, $E(\varepsilon_l)=0, E(\varepsilon_l^2)=\sigma_\varepsilon^2$, parameters ν, σ^2 are known, l is the number of iteration. The nonlinear equation from the model (3) are linearized relative to the variable $\mathbf{\Theta}_l$ using Taylor series with first approximation at the point $\widehat{\mathbf{\Theta}}_{l-1}$:

$$S(\mathbf{\Theta}_l)\approx S(\widehat{\mathbf{\Theta}}_{l-1})+\mathbf{S}'(\widehat{\mathbf{\Theta}}_{l-1})(\mathbf{\Theta}_l-\widehat{\mathbf{\Theta}}_{l-1})=d_{l-1,0}+\mathbf{d}_{l-1,1}\mathbf{\Theta}_l, \quad (4)$$

where $\mathbf{S}'(\widehat{\mathbf{\Theta}}_{l-1})$ – fir -order derivative of $S(\cdot)$ at the point $\widehat{\mathbf{\Theta}}_{l-1}$,

$$d_{l-1,0}=S(\widehat{\mathbf{\Theta}}_{l-1})-\mathbf{S}'(\widehat{\mathbf{\Theta}}_{l-1})\widehat{\mathbf{\Theta}}_{l-1},$$

$$\mathbf{d}_{l-1,1}=S'(\widehat{\mathbf{\Theta}}_{l-1})=\begin{pmatrix}S'(\widehat{A}_{l-1}) & S'(\Delta\widehat{f}_{l-1}) & S'(\widehat{\varphi}_{l-1})\end{pmatrix}_{1\times 3},$$

$$S'(\widehat{A}_{l-1})=\sum_{i=1}^{m}\frac{-\mathbf{Y}_i^T\mathbf{B}_i\boldsymbol{\rho}_{i,l-1}+\widehat{A}_{l-1}(I_i^2+J_i^2)}{S_i(\widehat{\mathbf{\Theta}}_{l-1})}$$

$$S'(\widehat{A}_{l-1})=\sum_{i=1}^{m}\frac{-\mathbf{Y}_i^T\mathbf{B}_i\boldsymbol{\rho}_{i,l-1}+\widehat{A}_{l-1}(I_i^2+J_i^2)}{S_i(\widehat{\mathbf{\Theta}}_{l-1})},$$

$$S'(\widehat{\varphi}_{l-1})=\widehat{A}_{l-1}\sum_{i=1}^{m}\frac{\mathbf{Y}_i^T\mathbf{C}_i\boldsymbol{\rho}_{i,l-1}}{S_i(\widehat{\mathbf{\Theta}}_{l-1})},$$

$$S'(\Delta\widehat{f}_{l-1})=2\pi T_c\widehat{A}_{l-1}\sum_{i=1}^{m}\frac{\mathbf{Y}_i^T\mathbf{C}_i\boldsymbol{\rho}_{i,l-1}i}{S_i(\widehat{\mathbf{\Theta}}_{l-1})},$$

$$\mathbf{B}_i=\begin{pmatrix}I_i & -J_i\\ J_i & I_i\end{pmatrix},\quad \mathbf{C}_i=\begin{pmatrix}J_i & I_i\\ -I_i & J_i\end{pmatrix},$$

$$\boldsymbol{\rho}_{i,l-1}=\begin{pmatrix}\cos(2\pi\Delta\widehat{f}_{l-1}T_ci+\widehat{\varphi}_{l-1}\\ \sin(2\pi\Delta\widehat{f}_{l-1}T_ci+\widehat{\varphi}_{l-1}\end{pmatrix}.$$

We write Tickhonov regularization to get estimations $\widehat{\mathbf{\Theta}}_l$ [18-20] using formulas (3, 4):

$$F(\mathbf{\Theta}_l,\widehat{\mathbf{\Theta}}_{l-1},...,\widehat{\mathbf{\Theta}}_0)=\sum_{l=1}^{L_0}\left(\frac{\left(me^{\nu+\frac{\sigma^2}{2}}-d_{l-1,0}-\mathbf{d}_{l-1,1}\mathbf{\Theta}_l\right)^2}{\sigma_\varepsilon^2}+\left\|\mathbf{\Theta}_l-\widehat{\mathbf{\Theta}}_{l-1}\right\|^2_{\mathbf{R}_l^{-1}}\right),$$

where $\mathbf{R}_l=E\left((\mathbf{\Theta}_l-\widehat{\mathbf{\Theta}}_{l-1})(\mathbf{\Theta}_l-\widehat{\mathbf{\Theta}}_{l-1})^T\right)$ is covariance matrix of extrapolation errors, σ_ε^2 is the error variance including the estimation error of the mathematical expectation with the sample mean and the error of Taylor approximation. We observe the estimation $\widehat{\mathbf{\Theta}}_l$ with the criterion. As the result, we get the expressions:

$$\widehat{\mathbf{\Theta}}_l=\widehat{\mathbf{\Theta}}_{l-1}+\mathbf{K}_l\left(me^{\nu+\frac{\sigma^2}{2}}-S(\widehat{\mathbf{\Theta}}_{l-1})\right),\ l=1,...,L_0 \quad (5)$$

where $\mathbf{K}_l=\mathbf{R}_l\mathbf{d}_{l-1,1}^T(\mathbf{d}_{l-1,1}\mathbf{R}_l\mathbf{d}_{l-1,1}^T+\sigma_\varepsilon^2)^{-1}$, $\mathbf{R}_l=\mathbf{G}_{l-1}+\sigma_\xi^2\mathbf{I}_{3\times 3}$, $\mathbf{G}_l\approx\mathbf{R}_l-\mathbf{K}_l\mathbf{d}_{l-1,1}\mathbf{R}_l$, initial conditions $\mathbf{G}_0=\sigma_\varepsilon^2\mathbf{I}_{3\times 3}$, $\widehat{\mathbf{\Theta}}_0=\begin{pmatrix}\widehat{A}_0 & \Delta\widehat{f}_0 & \widehat{\varphi}_0\end{pmatrix}^T=\begin{pmatrix}1 & 0 & 0\end{pmatrix}^T$ is from a priory information, $\mathbf{G}_l=E\left((\mathbf{\Theta}_l-\widehat{\mathbf{\Theta}}_l)(\mathbf{\Theta}_l-\widehat{\mathbf{\Theta}}_l)^T\right)$ is the covariance matrix of estimation errors.

As an alternative in the algorithm (5) $E(a_i)$ could be replaced by the median of a lognormal distribution $med=e^\nu$, because the median is tolerant to abnormal deviations in the sample of the observed process. Then we get the following expression for estimations:

$$\widehat{\mathbf{\Theta}}_l=\widehat{\mathbf{\Theta}}_{l-1}+\mathbf{K}_l\left(me^\nu-S(\widehat{\mathbf{\Theta}}_{l-1})\right),\ l=1,...,L_0. \quad (6)$$

One of the main advantages of algorithms (5) and (6) is their ability to utilize prior information about the distribution of the amplitude of the additive noise. This is a significant benefit because, in real-world conditions, signals are often affected by complex, non-Gaussian noise, for which standard methods may be less effective. If the noise distribution is known, these algorithms can adapt and more accurately account for the specific characteristics of the noise, leading to improved accuracy in estimating channel parameters.

This is particularly useful when the noise has a complex or impulsive nature, and its distribution differs from the Gaussian model. Taking into account prior information about the noise distribution allows the algorithms to estimate the communication channel parameters more precisely. More accurate parameter estimation reduces the probability of errors in information reception, which is especially important in high-order modulation schemes like 64-QAM or 256-QAM, where even small errors in parameter estimation can significantly degrade system performance.

One of the significant drawbacks is that signal processing is performed using the entire sample of the observed process. This means that in order to obtain an estimate of the unknown parameters, the full data sample must be accumulated before calculations can begin. This approach leads to increased latency in parameter estimation since the computation can only be performed after the entire signal sample has been collected. As a result, the time required to obtain parameter estimates becomes directly dependent on the duration of the test signal, limiting the application of such algorithms in real-time systems with strict speed requirements. An additional problem arises from the computational load associated with processing the data using algorithms (5) and (6).

Computational experiment. The computational simulation of algorithms (5), (6) is conducted using the following data: the test sequence is 64-QAM; $T_c=0.25$ us; sampling time is equal to the size T_c of test symbol I_i, J_i; the signal amplitude $A=3$, the signal phase including phase noise is defined by following expression $\varphi_i=\varphi_0+b_0\varepsilon_i+b_1\varepsilon_{i-1}$, where ε_i is white Gaussian noise with zero mathematical expectation and the variance σ_ε^2, b_0, b_1 are coefficients of the first-order moving average model and values are $\sigma_\varepsilon^2=3\cdot 10^{-4}$, $b_0=1$, $b_1=-0.1$ thus providing for MSE of phase noise is equal to one degree; φ_0 is the initial random phase which is distributed uniformly in the range $\begin{bmatrix}-\pi & \pi\end{bmatrix}$, the variance of dynamic system noise in the model (3) $\sigma_\xi^2=0$, $\sigma_\varepsilon^2=10^{-2}$; the number of implementations $N = 100$. Parameters of the lognormal distribution $\sigma^2 = 1, \nu = -3$ thus corresponds to the signal to noise ratio (SNR) $q_{\Pi} = 10\lg\left(\frac{P_N}{P_{\Pi}}\right) = 23$ dB (15.22 dB/bit)). The figure 1 shows dependencies the MSE of 64-QAM signal parameters on the quantity of algorithm iterations (5), (6), volume of observed process is $m = 300$ samples, the frequency shift $\Delta f = 500$ Hz, the signal to noise ratio $q_N = 10\lg\left(\frac{P_c}{P_N}\right) = 40$ dB (32.22 dB/bit)), $\sigma_{\mu}^2 = 10^{-4}$ and the signal to (noise + interference) effect ratio $q = 10\lg\left(\frac{P_S}{P_I + P_N}\right) - 10\lg 6 \cong 15$ dB/bit. The figure 2 with the same input data shows dependencies of the parameter's estimation MSE on volume sample m.

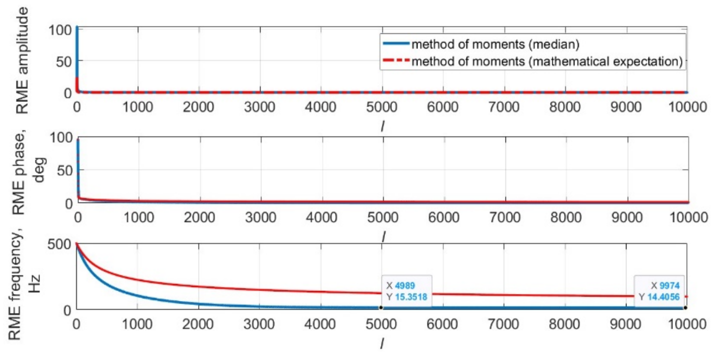


Fig. 1. The dependence of experimental MSE of 64-QAM signal parameters estimation with size *m* = 300 samples on the number of algorithm iterations

Figure 1 shows in the case with the number of iterations $L_0 = 5000$ MSE of the estimation using algorithms (5), (6), amplitude MSE_A, phase MSE_{φ} and frequency $MSE_{\Delta f}$ are equal:

$$MSE_{A6} = 0.018, MSE_{\varphi 6} = 0.24^{\circ}, MSE_{\Delta f6} = 15.35 \text{ Hz},$$

$$MSE_{A5} = 0.012, MSE_{\varphi 5} = 1.7^{\circ},$$

$$MSE_{\Delta f5} = 124.2 \text{ Hz, and with } L_0 = 10000 :$$

$$MSE_{A5} = MSE_{A6} = 0.0097,$$

$$MSE_{\varphi 6} = 0.22^{\circ}, MSE_{\Delta f6} = 14.4 \text{ Hz},$$

$$MSE_{\varphi 5} = 1.375^{\circ}, MSE_{\Delta f5} = 98.65\ Hz.$$

Figure 2 illustrates the method of moments based on the median allows to reach higher accuracy of the signal parameters estimation than the method which is using the mean value, especially with small samples of the observed process.

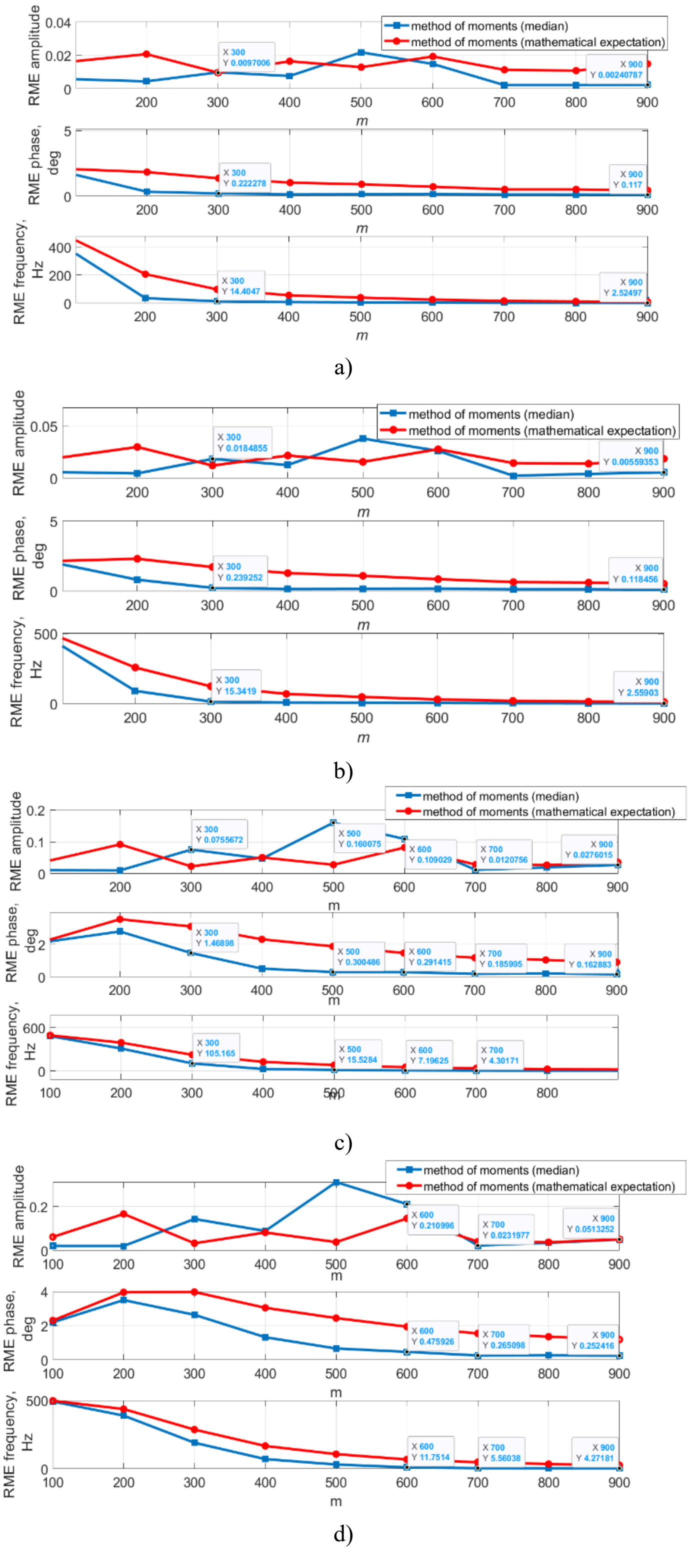


Fig. 2. The dependence of experimental MSE of 64-QAM signal parameters estimation on its size using recursive algorithms (5, 6) with $\Delta f = 500$ Hz,: $L_0 = 10000$ (a); $L_0 = 5000$ (b); $L_0 = 1000$ (c); $L_0 = 500$ (d)

The comparison of the algorithm (6) based on the method of moments (median) with the advanced Kalman filter is given below:

$$\widehat{\mathbf{\Theta}}_i = \widehat{\mathbf{\Theta}}_{i-1} + \mathbf{K}_i(\mathbf{Y}_i - \mathbf{\Phi}_i(\widehat{\mathbf{\Theta}}_{i-1})), i = 1,2,,,m \quad (7)$$

$$\mathbf{K}_i = \mathbf{P}_i\mathbf{D}_{1i}^T(\mathbf{D}_{1i}\mathbf{P}_i\mathbf{D}_{1i}^T + \mathbf{Q})^{-1},\ \mathbf{P}_i = \mathbf{\Gamma}_{i-1} + \mathbf{B},\ \mathbf{\Gamma}_i = \mathbf{P}_i - \mathbf{K}_i\mathbf{D}_{1i}\mathbf{P}_i,$$

where $\mathbf{\Gamma}_i = E(\mathbf{\Theta}_i - \widehat{\mathbf{\Theta}}_i)(\mathbf{\Theta}_i - \widehat{\mathbf{\Theta}}_i)^T$ is the filtration error covariation matrix, initial conditions: $\widehat{\mathbf{\Theta}}_0 = \begin{pmatrix}\widehat{A}_0 & \Delta\widehat{f}_0 & \widehat{\varphi}_0\end{pmatrix}^T = \begin{pmatrix}1 & 0 & 0\end{pmatrix}^T$, $\mathbf{\Gamma}_0 = \sigma_\gamma^2\mathbf{I}_{3\times3}$. Where $\mathbf{D}_{1i} = \mathbf{\Phi}_i'(\widehat{\mathbf{\Theta}}_{i-1})$ if derivative of the vector-function $\mathbf{\Phi}_i(\cdot)$ from the observed equation (1) at the point

$$\widehat{\mathbf{\Theta}}_{i-1} = \begin{pmatrix}\widehat{A}_{i-1} & \Delta\widehat{f}_{i-1} & \widehat{\varphi}_{i-1}\end{pmatrix}^T,\ \mathbf{B} = \begin{pmatrix}\sigma_{\xi A}^2 & 0 & 0\\ 0 & \sigma_{\xi\Delta f}^2 & 0\\ 0 & 0 & \sigma_{\xi\varphi}^2\end{pmatrix},$$

$\sigma_{\xi A}^2 = \sigma_{\xi\Delta f}^2 = 10^{-5}, \sigma_{\xi\varphi}^2 = 3\cdot10^{-4}$, $\mathbf{Q} = \sigma_\gamma^2\mathbf{I}_{2\times2}$.

In the end of the algorithm (7) we made recalculating estimations of the frequency shift and the phase as follows:

$$\begin{pmatrix}\Delta\widehat{f}\\ \widehat{\varphi}\end{pmatrix} = \left(\mathbf{F}^T\mathbf{F}\right)^{-1}\mathbf{F}^T\widehat{\boldsymbol{\psi}}, \quad (8)$$

where

$$\widehat{\boldsymbol{\psi}} = \begin{pmatrix}\widehat{\psi}_k\\ \widehat{\psi}_{k+1}\\ \vdots\\ \widehat{\psi}_m\end{pmatrix}_{(m-k+1)\times1},\ \mathbf{F} = \begin{pmatrix}2\pi T_c k & 1\\ 2\pi T_c(k+1) & 1\\ \vdots & \vdots\\ 2\pi T_c(m+1) & 1\end{pmatrix}_{(m-k+1)\times2},$$

$\widehat{\psi}_i = 2\pi\Delta\widehat{f}_i T_c i + \varphi_i$, $i = k, k+1, ..., m$. The initial time moment is $k = 40$ in the algorithm (8). The procedure (8) was conducted due to uncertainties of the equations system (1) with a view in further realize quasicoherent signal reception. Values of MSEs of the 64-QAM signal parameters estimation using algorithms (6) and (7), (8) are shown in the Table 1.

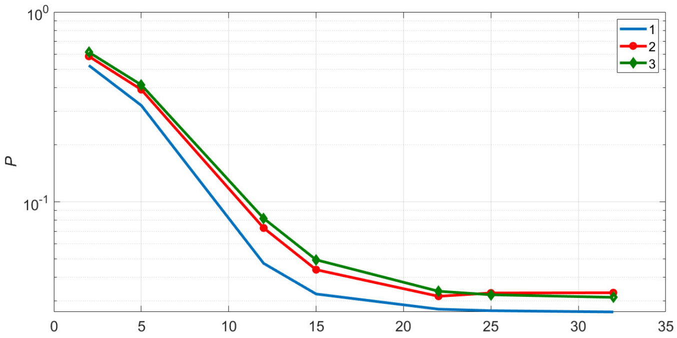
Table 1

MSE estimation of the 64-QAM signal's amplitude, phase and frequency shift using algorithms (6) and (7, 8) with the ration signal / (noise + interference effect) $q_{IE} = 15$ dB/bit

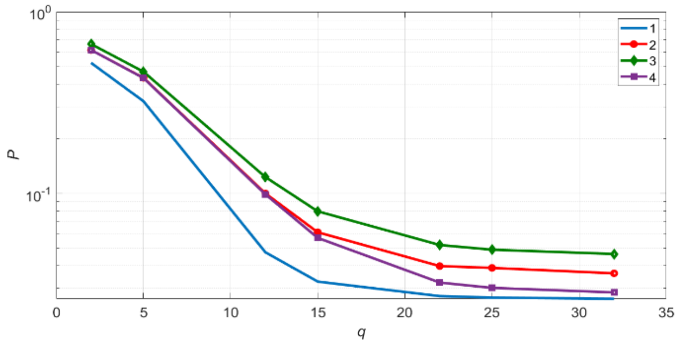
Method	MSE_A	MSE_φ, deg	$MSE_{\Delta f}$, Hz
m=300			
Advanced Kalman filter with recalculating estimations (7, 8)	$1.7\cdot10^{-2}$	0.5	28.19
Method of moments (median) (6), $L_0 = 5000$	$1.8\cdot10^{-2}$	0.24	15.34
m=900			
Advanced Kalman filter with recalculating estimations (7, 8)	$1.5\cdot10^{-2}$	0.25	4.99
Method of moments (median) (6), $L_0 = 5000$	$5.6\cdot10^{-3}$	0.12	2.56

Table 1 demonstrates that the accuracy of estimating signal parameters using the method of moments is superior compared to the advanced Kalman filter. The MSE of phase estimation and frequency shift, when applying algorithms (7) and (8), is approximately twice as large as the MSE obtained when using procedures defined by algorithm (6). This indicates a noticeable performance difference in favor of the method of moments.

Figure 3 presents the experimental noise immunity curves for 64-QAM signal reception in the presence of both additive white Gaussian noise (AWGN) and interference. The interference is modeled with a lognormal amplitude distribution and a uniform phase distribution. The figure compares the results using algorithms (6) and (7, 8) testing sequence size $m = 500$ in terms of the number of symbols. The curves show how the performance of the signal reception varies under these noise conditions, highlighting the robustness of the algorithms under various interference scenarios.



a)



b)

Fig. 3. The experimental error per symbol dependency of the SNR per bit q_N when receiving a 64-QAM signal using different algorithms of signal parameters estimation: values of parameters are known except for phase noise – 1; a: m = 500, n = 3000, the method of moments (median) (6), $L_0 = 1000$ – 2; the method of the nonlinear filtering (7, 8) – 3; б: $m = 500, n = 4500$, the method of moments (median) (6), $L_0 = 1000$ – 2, the method of the nonlinear filtering (7, 8) – 3, method of moments (6), $L_0 = 5000$ – 4.

The Figure 3 provides data about using the advanced Kalman filter and using the method of moments for the communication channel parameters estimation give similar results when $m = 500, n = 3000$ $(\frac{n}{m} = 6)$.

The case with $n = 4500$ ($\frac{n}{m} = 9$), which corresponds to an increase in the size of information signal to 50%, energy payoff of algorithm (6) over (7), (8) is from 0.5 to 4 dB/bit with the size of procedure iterations (6), based on the method of moments, $L_0 = 1000$ and to 6 dB/bit with $L_0 = 5000$ with SNR 5-22 dB/bit.

Computational complexity. The analyze the number of arithmetic operations N_{OP} for algorithms (6) and (7), (8) is shown in Table 2.

Table 2

Computational complexity of communication channel signal parameters estimation algorithms based on the recursive nonlinear filtering and on the method of moments

Algorithm	N_{OP}
Advanced Kalman filter (7) Recalculating estimations (8)	$150m$ $16m - 16k + 27$
Method of moments (6)	$(62m + 45)L_0$

The complexity of procedures (6) depends on the number of iterations L_0. For large values L_0 $N_{OP(6)} \gg N_{OP(7),(8)}$.

The method of moments relies heavily on iterative calculations. Its computational complexity is directly proportional to the number of iterations required for convergence. Each iteration involves performing several arithmetic operations based on the size of the input data and the structure of the algorithm. As the number of iterations increases, the overall computational complexity grows significantly. Therefore, in scenarios where a large number of iterations are required to reach an acceptable level of accuracy, the computational complexity of the method of moments can become substantially higher. In fact, when the iteration count becomes large, the method of moments can surpass the Kalman filter in terms of computational cost.

On the other hand, the advanced Kalman filter has a computational complexity that depends primarily on the size of the sample, specifically the number of data points or observations used. Unlike the method of moments, the Kalman filter does not involve iterative steps that affect its overall complexity in the same way. Instead, its complexity is mainly driven by matrix operations such as multiplication, inversion, and updates, which are performed at each time step. These operations depend on the dimensions of the state vector and the observation matrix but are relatively fixed per time step, making the computational load predictable and bounded by the sample size.

Conclusion

1) The application of the median in the method of moments yields a lower MSE in the estimation of communication channel signal parameters compared to using the mean value. This is due to the fact that the median is less sensitive to outliers in the random process, which is a critical advantage in scenarios where noise or other unexpected fluctuations might distort the signal. While the difference in estimation accuracy between the median and mean decreases as the number of signal samples increases, the median still tends to provide a more robust estimation, particularly in environments with significant noise or nongaussian interference. This highlights the importance of considering robust statistical techniques in the presence of non-ideal conditions that can affect parameter estimation in real-world communication channels.

2) The computational experiment demonstrates that the method of moments, particularly in estimating the phase and frequency shift of a 64-QAM signal, achieves an accuracy that is approximately 2 times higher than that of the advanced Kalman filter which leads to power payoff from 0.5 to 6 dB/bit. This improvement in estimation precision can directly impact system performance, reducing error rates and enhancing the overall reliability of signal reception.

3) The disadvantage of the method of moments is its complexity. This higher accuracy comes at the cost of increased computational power, as the method of moments requires more complex processing compared to the Kalman filter. This trade-off between accuracy and computational efficiency should be carefully considered when implementing these algorithms in practical systems, particularly in scenarios with limited processing resources or where real-time operation is required.

The computational complexity of method of moments grows with the number of iterations. For a small number of iterations, the method may be efficient, but as the iteration count increases, the complexity can surpass that of the Kalman filter, especially in high-precision scenarios. Kalman filter's computational complexity is determined primarily by the sample size and remains relatively stable since it does not depend on the number of iterations. The cost is governed by matrix operations, which scale based on the size of the state vector and the observation model.

One potential area for future research is the development of hybrid estimation algorithms that combine the strengths of both the method of moments and Kalman filtering. Such an approach could utilize the method of moments for initial coarse estimation or in scenarios with high noise levels, followed by Kalman filtering for fine-tuning the results in a computationally efficient manner. This could help to mitigate the disadvantages of each method when used alone, offering a balanced solution that combines accuracy and efficiency.

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МЕТОД МОМЕНТОВ В ЗАДАЧЕ ОЦЕНКИ ПАРАМЕТРОВ КАНАЛА СВЯЗИ

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Аннотация

В статье представлен синтез алгоритма оценки параметров канала связи на основе наблюдаемого сигнала M-QAM с известной информационной последовательностью. Предлагаемый алгоритм использует метод моментов, выраженный в виде функционала Тихонова А.Н., который позволяет эффективно оценивать различные параметры канала. Эти параметры включают амплитуду, фазу и сдвиг частоты принятого сигнала. Фазовый шум также включен в фазовую модель для обеспечения более точного отражения реальных условий канала. Процесс оценки проводился в условиях аддитивного белого гауссовского шума (AWGN), а также в сценариях, где шум следовал логнормальному распределению вероятностей. Предполагалось, что фаза следует равномерному распределению. Для оценки производительности предлагаемого алгоритма был проведен вычислительный эксперимент, сосредоточенный на точности оценки параметров для сигнала 64-QAM. Результаты, полученные с использованием этого метода, сравнивались с результатами, достигнутыми с помощью расширенной фильтрации Калмана, известного подхода к оценке параметров в системах связи. Выполнен грубый анализ вычислительной сложности алгоритмов, сравнивающий метод моментов с рекурсивными нелинейными алгоритмами фильтрации. Экспериментальные кривые, отображающие помехоустойчивость приема сигнала 64-QAM, были получены с использованием как синтезированного алгоритма, так и усовершенствованного метода фильтрации Калмана. Эти результаты дают ценную информацию об эффективности и практической осуществимости предлагаемого подхода к оценке параметров в каналах связи, особенно в шумных средах. Для дальнейшего подтверждения надежности предлагаемого алгоритма в моделировании были протестированы различные уровни шума и условия канала. Это позволило провести комплексную оценку адаптивности и точности алгоритма при различных отношениях сигнал/шум (SNR), что гарантирует его применимость в широком диапазоне практических сценариев связи.

Ключевые слова: *метод моментов, расширенный метод наименьших квадратов (LMS), расширенная фильтрация Калмана, оценка параметров канала связи, AWGN, логнормальный шум, сигнал M-QAM.*

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