RESOLUTION TIME THEORY IN THE TOPIC OF BROADBAND COMMUNICATIONS. ALGORITHM FOR DATA DEPENDENT JITTER AND CAPACITY ESTIMATIONS WITH POLYNOMIAL TIME EXECUTION

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At present, due to the constant growth of the volume of transmitted information, more and more attention is paid to the issues of the possibility of increasing the transmission speed due to the use of the transmission mode "above the Nyquist rate". Despite the relevance of this research topic, due to rather large mathematical difficulties, it is difficult to obtain significant results. One of the theories that allows for a breakthrough is the rapidly developing theory of resolution time developed for phase radio engineering data transmission systems and information-measuring optoelectronic systems with PAM-signals with square pulses. This paper presents a novel method for capacity and resolution time estimations with polynomial computational complexity that does not depend on the size of the channel alphabet of PAM-n-signals and is determined only by the effective memory value. The shape of pulse of PAM-n-signals has arbitrary form and amplitudes take only possitive values. The method also allows estimating the time limiting possible deviations due to the sampling error in time, at which the information about the channel symbol will be read error-free or with a given error probability.

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Introduction

At that moment, serial data transmission systems with pulse amplitude modulation signals (PAM-signal) are widely used in the following areas in wired communication systems (local area networks, storage area networks, wide-area networks and etc.); 2) digital television. While the serial link transceiver circuits have kept pace with the increase in processor speed, the physical interconnect between the devices has changed very little. However, signal attenuation, dispersion, and connector reflections limit the capacity of these links.

Recent protocols that support high-speed backplane communication such as InfiniBand, Gigabit Ethernet, Thunderbolt and have been adapted to consider the signal integrity issues in high-speed backplanes [1]. But further development of these standards leads to significant capacity limitations caused by deterministic jitter caused by data transmission [2].

This fact is proved by the fact that previous decade (2000-2010 ss) was concerned as time when a drastic change in the design of high-speed serial links is observed. Since Silicon fabrication technology has produced smaller, faster transistors, transmission line interconnects between backplanes have not substantially improved [1]. This has led to the fact that over the past decade there has been some slowdown in the development of this information transfer technology, with forecasts to overcome it in the period from 2025-2030.

This article is devoted to the developing of a method with polynomial computational complexity for study specifically deterministic jitter in mentioned above systems. The creation of this method should provide a solution to the following issues: 1) predicting the signal integrity problems that occur in wireline channels in serial links caused by data dependent jitter without restrictions on the constellation configuration and complex frequency response of the frequency selective communication channel and pulse shape; 2) channel capacity and resolution time estimation; 3) time limiting possible deviations estimation due to the sampling error in time.

In contrast to the previously obtained results in the J.F. Buckwalter's, G.I. Il'in's and Yu.E.Polski works [1, 3-6] in this paper the obtained method has no limitations on: 1) complex frequency response of such channel, 2) on the size of signal constellations; 3) on the form pulse shape. In addition, a solution was also obtained for capacity estimation for such communication systems using the linear receiver.

1. Math model of frequency selective channel with PAM-signals. Problem Statement

According to the monograph [7] the block diagram of the transmission path has the following form for such channel can be represeted as follows (see Fig. 1)

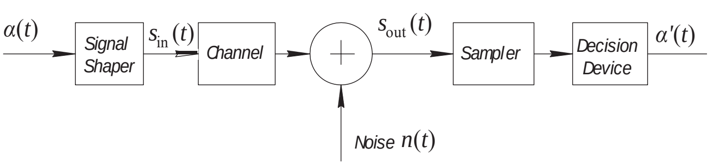

Fig. 1. Structural diagram of the transmission path

At the input of the signal shaper, a message arrives that can be represented as a lattice function $\alpha(t)$ in the following form

$$\alpha(t) = \sum_{r=1}^{l} M_r \delta\left(t-(r-1)\tau_s\right), \tag{1}$$

where $\delta(t)$ is Dirac delta function; l is the number of symbols in information sequence; $\tau_s = 1/V_0$ is a clock interval, determines the transmission rate V_0, M_r is the weight of Dirac delta function for the depending on information content of the r-th signal element. Each $M_r \in \mathbf{M}$, where $\mathbf{M} = \left\{M_{\text{sc}_k}\right\}_{k=1}^{n} = \left\{M_{\text{sc}_k} = k\Delta M_{\text{st}} + M_{\text{sh}}, k=\overline{1,n}\right\}$ is a signal constellation (SC) of PAM-n-signal; ΔM_{st} is a step between adjacent values of SC; M_{sh} is amplitude shift of SC. It should be noted that according to paper [7] that each M_r, $r=\overline{0,l}$ takes equiprobable and independent one value from set $\mathbf{M}$.

Signal shaper forms the PAM-n-signal. This action can be presented in the following form

$$s_{\text{in}}(t) = \sum_{r=1}^{l} M_r g_{\text{sh}}\left(t-(k-1)\tau_s\right), \tag{2}$$

where $g_{\text{sh}}(t)$ is a cutting function of signal shaper, signal shaper pulse response [7].

When a signal (2) is applied to a frequency-selective channel the output signal can be described as follows

$$s_{\text{out}}(t) = s_{\text{in}}(t) * g_{\text{ch}}(t), \tag{3}$$

where $g_{\text{ch}}(t)$ is impulse response of channel and * is convolution operation.

It should be noted that half duration of $g_{\text{sh}}(t)$ should be such that it is the duration during which the transient response reaches a level of 0.9 from its stationary value, otherwise we need to check if rule (10) is true for $d = 1$.

In this paper the sampler performs the following operation according results of [7]

$$s_{\text{sp}}(t) = \sum_{r=1}^{l} s_{\text{out}}(r\tau_s)\left[1\left(\frac{t-(r-1)\tau_s}{\tau_s}\right) - 1\left(\frac{t-r\tau_s}{\tau_s}\right)\right], \tag{4}$$

where $1(t)$ is a Heaviside function.

Due to various unfavorable factors, the sampler can carry out the read the value about the information parameter with some time sampling error $\Delta T_s \in \mathbb{R}$. Since the expression (4) should be rewrite in the following form

$$s_{\text{sp}}(t) = \sum_{r=1}^{l} s_{\text{out}}(r\tau_s + \Delta T_s)\left[1\left(\frac{t-(r-1)\tau_s}{\tau_s}\right) - 1\left(\frac{t-r\tau_s}{\tau_s}\right)\right] \tag{5}$$

Decision device reconstructs each $d=\overline{1,l}$ channel symbol in accordance with the rule

$$M_{rec}(d\tau_s) = M_p\big|_{p=p'}, \tag{6}$$

where

$$p' \in \overline{1,m}:\ f(p',d) = \min_{p\in\overline{1,m}} \left| H_{\text{ms}}(d\tau_s) - M_p \right|;$$

$$H_{\text{ms}}(t) = \frac{s_{\text{out}}(t) + n(t)}{k_{\text{norm}}} = s_{\text{out.norm}}(t) + n_{\text{norm}}(t)$$

is measured signal on the output of channel after normalization operation; k_{norm} is normalization coefficient, its value estimation is made by least mean square method using training sequence and the following equalities must be hold $\hat{M}_{sc_1} \simeq M_{sc_1}$ and $\hat{M}_{sc_n} \simeq M_{sc_n}$, where $\hat{M}_{sc_1}$ and $\hat{M}_{sc_n}$ are the math expectation of normalized amplitudes of M_{sc_1} and M_{sc_n}, respectively, on the input of decision device. The normalization procedure of the $s_{\text{out}}(t)$ occurs before the sampling procedure in sampler.

In this paper we use two type of noise is used:

1st type of noise $n_{\text{norm}}(t)$ is a stationary random process; each of its section is a random variable, whose probability density function $f_n(N)$ is determined according to the following expression

$$f_n(N) = \begin{cases} \dfrac{1}{2\Delta}, & N \in [-\Delta;\Delta] \\ 0, & N \notin [-\Delta;\Delta] \end{cases}; \qquad (7)$$

where Δ is the absolute value of the limiting measurement errors caused by sampler. Its fiducial value is defined as $\Delta_0 = \Delta / \Delta M_{\text{st}}$

2nd type of noise $n_{\text{norm}}(t)$ is a stationary random process with null math expectation; each of its section is a random variable, whose probability density function $f_n(N)$ is determined according to the following expression

$$f_n(N) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-0.5 \left(\frac{N}{\sigma} \right)^2 \right); \qquad (8)$$

where σ^2 is variance.

Based on the results of the work [3] the variance σ^2 can be estimated by using $\sigma = \Delta / C_{\text{t}}$, where $C_{\text{t}} = F^{-1}(1 - P_{\text{a}})$; $F^{-1}(.)$ is the inverse function of the standard normal probability distribution and P_{a} is an anomalous error probability which determines the probability of exceeding the value $\pm\Delta$.

Based on (6) the recovered (received) signal we have the following form

$$\alpha'(t) = \sum_{r=1}^{l} M_{\text{rec}} \delta(t - r\tau_s). \qquad (9)$$

It is obviously that in order to correctly recover the values of each channel symbol of the data sequence, it is required that the selected symbol duration channel symbol τ_s with time error sampling ΔT_s satisfies the set of values $\tau_s + \Delta T_s \in T$, where for a set T the following inequality is correct

$$\left| H_{\text{ms}}(d\tau_s + \Delta T_s) - M_d \right| \le Q_{\text{A}} \Leftrightarrow$$
$$\Leftrightarrow \left| \Delta_{\text{set}}(dT - (d-1)\Delta T_s) + n_{\text{norm}}(dT - (d-1)\Delta T_s) \right| \le Q_{\text{A}}, \qquad (10)$$

where $\Delta_{\text{set}}(d\tau_s + \Delta T_s) = s_{\text{out.norm}}(d\tau_s + \Delta T_s) - M_d$ is the settling error for d-th channel symbol at the moment $d\tau_s + \Delta T_s$; Q_{A} is the limit by which it can differ $H_{\text{ms}}(d\tau_s + \Delta T_s)$ from the value d-th channel symbol, at which ensures the correct recovery of each channel symbol of the transmitted sequence; while it is obvious $Q_{\text{A}} < 0.5\Delta M_{\text{st}}$, $Q_{\text{A}} \to 0.5\Delta M_{\text{st}}$.

According to results presented in papers [8-18] and expression (10) the set T can be determined in the form

$$T = \bigcup_{i=1}^{W} [t_{\text{w.st.i}}; t_{\text{w.end.i}}] \qquad (11)$$

Here $t_{\text{w.st.i}}$, $t_{\text{w.end.i}}$ are symbol duration at which it starts and ends i-th transparency window, respectively, that is $[t_{\text{w.st.i}}; t_{\text{w.end.i}}] \subset T$; $i = \overline{1,W}$ and W are the number of the transparency window and their amount, respectively.

Obviously, at $l \to \infty$ the set T determines the resolution time $t_{res} = \{t_{\text{w.st.i}}\}_{i=1}^{W} \cup \{t_{\text{w.end.i}}\}_{i=1}^{W}$ for the considered model, because:

- when using a set of T symbol durations for the $\forall d = \overline{1,l}$ -th symbol of the sequence and $l \to \infty$ the following relation will be true

$$\left| \Delta_{\text{set}}(dT - (d-1)\Delta T_s) + n_{\text{norm}}(dT - (d-1)\Delta T_s) \right| \le$$
$$\le \Delta_{\max}(dT) + \Delta < 0.5\Delta M_{\text{st}}, \qquad (12)$$

where

$$\Delta_{\max}(dT \mp (d-1)\Delta T_s) = \max \left| \Delta_{\text{set}}(dT \mp (d-1)\Delta T_s) \right|.$$

Here, the settling error is maximized over all possible realizations of symbol values in the transmitted sequence with the number of symbols equal d for each channel symbol duration value. Obviously, that for the output PAM-n-signal for given values of Q_{A} и Δ we can specify the value of magnitude admissible settling error Δ_{adm}, which is determined using the following system

$$\left. \begin{array}{c} \Delta_{\text{adm}} = Q_{\text{A}} - \Delta \\ \forall d = \overline{1,l},\ l \to \infty : \Delta_{\max}(dT \mp (d-1)\Delta T_s) \le \Delta_{\text{adm}} \end{array} \right\} \qquad (13)$$

Wherein $t_{\text{w.st.i}}$ can be defined as follows

$$\exists d : \Delta_{\max}\left(d\overset{\circ}{U}(t_{\text{w.st.i}} - 0, \varepsilon) \right) > \Delta_{\text{adm}}, \Delta_{\max}(dt_{\text{w.st.i}}) = \Delta_{\text{adm}}, \qquad (14)$$

and $t_{\text{w.end.i}}$ as follows

$$\exists d :\ \Delta_{\max}\left(d\overset{\circ}{U}(t_{\text{w.end.i}} + 0, \varepsilon) \right) > \Delta_{\text{adm}},\ \Delta_{\max}(dt_{\text{w.end.i}}) = \Delta_{\text{adm}}. \qquad (15)$$

Here and below $\overset{\circ}{U}(\ .\ )$ is a ε-punctured neighborhood.

According to results of papers [8-15] the general group of capacity estimations is determined as follows

$$C = t_{\text{res}}^{-1} \log_2 n \tag{16}$$

Based on the expression (16) and $t_{res} = \{t_{\text{w.st.i}}\}_{i=1}^{W} \cup \{t_{\text{w.end.i}}\}_{i=1}^{W}$ the set of capacity estimations can be represented as follows:

$$C_i^{'} = C\Big|_{t_{\text{res}}=t_{\text{w.st.i}}};\ C_i^{''} = C\Big|_{t_{\text{res}}=t_{\text{w.end.i}}}. \tag{17}$$

Here, $C_i^{'}$ and $C_i^{''}$ are estimations of the upper and the low limit of the capacity, respectively, for the i-th "transparency window".

Analyzing expressions (1) – (15) and taking into account the results of the papers [8-10] the *problem statement* can be formed in the following form:

1) combinations of transmitted channel symbols must be found for which the settling time is the greatest;

2) a rule for estimating the required number of sequence symbols is defined, under which, due to the validity of the transposition principle for linear frequency-selective communication channels, an estimate of the resolution time with given accuracy will be provided.

3) the rule for determining the limit range ΔT_s should be formulated.

2. The Resolution Time Theory for PAM-n-signal and Frequency Selective Channel. Problem Solution

2.1 Polynomial Complexity Algorithm to Estimate the Greatest Settling Time

Taking into account the results of the theory of resolution time for radio engineering data transmission systems [8-18] and expression (12) we need to solve the following problems:

1st problem is to get the expression for estimation τ_s when inequality (12) is true and subject to fulfillment $\Delta T_s = 0$;

2nd problem is to synthesis the rule to estimate ΔT_s for each value τ_s, at which the equation (12) is true.

The solution of 1st problem has the following form. First of all, we need to obtain the expression for greatest settling time estimation $t'_d = \{t'_{\text{w.st.i,d}}\}_{i=1}^{p} \cup \{t'_{\text{w.end.i,d}}\}_{i=1}^{p}$ on the output of channel for PAM-n-signal that consists of d symbols, when symbol duration of each symbol is t'_d. Here $t'_{\text{w.st.i,d}}$ and $t'_{\text{w.end.i,d}}$ are symbol time duration at which i-th transperency window starts and ends, p is amount of transperency windows for greatest settling time t'_d.

For this, it is first necessary to obtain expressions for determining the settling error $\Delta_{\text{set}}(d\tau_s)$ for d-th symbol. In this case the expression (3) convenient to represent in the following form

$$s_{\text{out}}(t) = \sum_{r=1}^{l} M_r P\left(t - (r-1)\tau_s\right), \tag{18}$$

where $P(t)$ is the response of a frequency selective channel on the pulse with a unit amplitude generated by a shaper.

Let us transform the expression (18) using the following substitutions $l = d$, $t = l\tau_s = d\tau_s$ and $s_{\text{out}}(d\tau_s) = M_d + \Delta_{\text{set}}(d\tau_s)$. As the result we get the following expression

$$M_d + \Delta_{\text{set}}(d\tau_s) = \sum_{r=1}^{d} M_r P\left((d-r+1)\tau_s\right) = \sum_{r=1}^{d} M_r P_{r,d}(\tau_s), \tag{19}$$

where $P_{r,d}(\tau_s) = P\left((d-r+1)\tau_s\right)$.

From equality (19) follows the desired expression for the settling error for d-th channel symbol

$$\Delta_{\text{set}}(d\tau_s) = \sum_{r=1}^{d} M_r P_{r,d}(\tau_s) - M_d = \\ = \sum_{r=1}^{d-1} M_r P_{r,d}(\tau_s) - M_d\left[P_{d,d}(\tau_s) - 1\right], \tag{20}$$

To determine the condition when the following relation $\Delta_{\max}(dt'_d) = \Delta_{\text{adm}}$ is true, let us find the conditions at which the expression (20) reaches its extremums assuming that $\tau_s = t'_d$.

First of all, we solve the following problem

$$\left.\begin{aligned} &\frac{\partial \Delta_{\text{set}}(dt'_d)}{\partial M_1} = 0 \\ &\frac{\partial \Delta_{\text{set}}(dt'_d)}{\partial M_2} = 0 \\ &\quad\vdots \\ &\frac{\partial \Delta_{\text{set}}(dt'_d)}{\partial M_d} = 0 \end{aligned}\right\}. \tag{21}$$

After a series of simple transformations, system (21) will take the form

$$\left.\begin{aligned} &P_{1,d}(t'_d) = 0 \\ &P_{2,d}(t'_d) = 0 \\ &\quad\vdots \\ &P_{d,d}(t'_d) = 1 \end{aligned}\right\}. \tag{22}$$

In general case the solution (22) doesn't exist, because:

1) A solution to the system will only exist when using a raised cosine filter, which frequency response significantly differs from such response for the real frequency-selective communication channel. In this case, the duration of the symbol and sampling must meet the Nyquist criteria for ISI free reading information.

2) In another case the solution doesn't exist since nowadays Faster that Nyquist regime is used or a mode close in speed to it, in which the reading of information about the channel symbol is made in the presence of ISI.

It follows from the system (22) that the signal amplitude does not affect the reaching extremums; therefore extremums occur at the boundaries of the signal constellation. Therefore, for sequence consisting of d-th symbols we need solve $2^d - 1$ equations to obtain t'_d due to the same number of amplitude combinations that which corresponds to the sub exponential time complexity of the algorithm.

To further simplify the computational complexity of the algorithm of t'_d calculation, consider the expression (20) rewriting it in the following way

$$\Delta_{set}(d\tau_s)=\underbrace{\sum_{k_1=1}^{d-1}M_{k_1}\left|P_{k_1,d}(\tau_s)\right|\chi_+\left(P_{k_1,d}(\tau_s)\right)+M_d\left|\left[P_{d,d}(\tau_s)-1\right]\right|\chi_+\left(P_{d,d}(\tau_s)-1\right)}_{\text{positive sum}}$$
$$+\underbrace{\sum_{k_2=1}^{d-1}M_{k_2}\left|P_{k_2,d}(\tau_s)\right|\chi_-\left(P_{k_2,d}(\tau_s)\right)+M_d\left|\left[P_{d,d}(\tau_s)-1\right]\right|\chi_-\left(P_{d,d}(\tau_s)-1\right)}_{\text{negaitve sum}} \quad (23)$$

Where

$\chi_+(x)=\mathrm{sgn}(\mathrm{sgn}[x]+1)$; $\chi_-(x)=\mathrm{sgn}(\mathrm{sgn}[x]-1)$; sgn(.) is a signum function.

It is obvious that to determine the conditions when $\forall\tau_s,\forall d:$ $\left|\Delta_{set}(d\tau_s)\right|\to\max$, we need that one of the following conditions set come true:

1st set

$$\forall\tau_s,\forall d:\sum_{k_1=1}^{d-1}M_{k_1}\left|P_{k_1,d}(\tau_s)\right|\chi_+\left(P_{k_1,d}(\tau_s)\right)+ +M_d\left|\left[P_{d,d}(\tau_s)-1\right]\right|\chi_+\left(P_{d,d}(\tau_s)-1\right)\to\max; \quad (24)$$

$$\forall\tau_s,\forall d:\left|\sum_{k_2=1}^{d-1}M_{k_2}\left|P_{k_2,d}(\tau_s)\right|\chi_-\left(P_{k_2,d}(\tau_s)\right)+ +M_d\left|\left[P_{d,d}(\tau_s)-1\right]\right|\chi_-\left(P_{d,d}(\tau_s)-1\right)\right|\to\min. \quad (25)$$

2nd set

$$\forall\tau_s,\forall d:\sum_{k_1=1}^{d-1}M_{k_1}\left|P_{k_1,d}(\tau_s)\right|\chi_+\left(P_{k_1,d}(\tau_s)\right)+ +M_d\left|\left[P_{d,d}(\tau_s)-1\right]\right|\chi_+\left(P_{d,d}(\tau_s)-1\right)\to\min; \quad (26)$$

$$\forall\tau_s,\forall d:\left|\sum_{k_2=1}^{d-1}M_{k_2}\left|P_{k_2,d}(\tau_s)\right|\chi_-\left(P_{k_2,d}(\tau_s)\right)+ +M_d\left|\left[P_{d,d}(\tau_s)-1\right]\right|\chi_-\left(P_{d,d}(\tau_s)-1\right)\right|\to\max. \quad (27)$$

The solution for the first set is $\forall M_{k_1}=M_{sc_n}$, $M_d=M_{sc_n}$ (problem (24)) and $\forall M_{k_2}=M_{sc_1}$, $M_d=M_{sc_1}$ (problem (25)). For second set solution has the following form: $\forall M_{k_1}=M_{sc_1}$, $M_d=M_{sc_1}$ (problem (26)) and $\forall M_{k_2}=M_{sc_n}$, $M_d=M_{sc_n}$ (problem **Ошибка! Источник ссылки не найден.**).

Considering the above obtained results expression for $\Delta_{max}(d\tau_s)$ will take the form

$$\Delta_{max}(d\tau_s)=\left|S_+(\tau_s)1\left(\left|S_+(\tau_s)\right|-\left|S_-(\tau_s)\right|\right)+ +S_-(\tau_s)1\left(\left|S_-(\tau_s)\right|-\left|S_+(\tau_s)\right|\right)\right|\Phi(\tau_s), \quad (27)$$

where

$$\Phi(\tau_s)=1-\frac{1\left(\left|S_+(\tau_s)\right|-\left|S_-(\tau_s)\right|\right)\times1\left(\left|S_-(\tau_s)\right|-\left|S_+(\tau_s)\right|\right)}{2};$$

$1(t)$ is Heaviside step function;

$$S_+(\tau_s)=\sum_{r=1}^{d-1}\left|P_{r,d}(\tau_s)\right|\left[M_{sc_n}\chi_+\left(P_{r,d}(\tau_s)\right)+M_{sc_1}\chi_-\left(P_{r,d}(\tau_s)\right)\right]+ +\left|\left[P_{d,d}(\tau_s)-1\right]\right|\left(M_{sc_n}\chi_+\left(P_{r,d}(\tau_s)-1\right)+M_{sc_1}\chi_-\left(P_{r,d}(\tau_s)-1\right)\right);$$

$$S_-(\tau_s)=\left|\sum_{r=1}^{d-1}\left|P_{r,d}(\tau_s)\right|\left[M_{sc_1}\chi_+\left(P_{r,d}(\tau_s)\right)+M_{sc_n}\chi_-\left(P_{r,d}(\tau_s)\right)\right]+ +\left|\left[P_{d,d}(\tau_s)-1\right]\right|\left(M_{sc_1}\chi_+\left(P_{r,d}(\tau_s)-1\right)+M_{sc_n}\chi_-\left(P_{r,d}(\tau_s)-1\right)\right)\right|.$$

Using expression (27) the desired equation $\Delta_{max}\left(dt'_d\right)=\Delta_{adm}$ used to evaluate t'_d takes the form

$$\Delta_{adm}=\left|S_+(t'_d)1\left(\left|S_+(t'_d)\right|-\left|S_-(t'_d)\right|\right)+ +S_-(t'_d)1\left(\left|S_-(t'_d)\right|-\left|S_+(t'_d)\right|\right)\right|\Phi(t'_d). \quad (28)$$

Analyzing (28) we can conclude that we have the polynomial time complexity of the algorithm to estimate t'_d, because we need to solve only one equation with $2d$ polynomials.

The solution of 2nd problem problem has the following form. To obtain the deviation set of symbol time sampling at which inequality (12) is true we have to solve problem similar to first problem.

First of all, for this we need to obtain an expression for settling error estimation for d-th symbol $\Delta T_s\neq0$ and symbol duration $\tau'_{s_d}\in\bigcup_{i=1}^{p}\left[t'_{w.st.i,d};t'_{w.end.i,d}\right]$. Let us transform the expression (18) using the following substitutions $l=d+1$, $t=d\tau'_{s_d}+\Delta T_s$ and $s_{out}\left(d\tau'_{s_d}+\Delta T_s\right)=M_d+\Delta_{set}\left(d\tau'_{s_d}+\Delta T_s\right)$. As the result we get the following expression

$$s_{out}\left(d\tau'_{s_d}+\Delta T_s\right)=M_d+\Delta_{set}\left(d\tau'_{s_d}+\Delta T_s\right)= =\sum_{r=1}^{d+1}M_rP\left(d\tau'_{s_d}+\Delta T_s-(r-1)\tau_s\right). \quad (29)$$

After some simple transformations the expression for $\Delta_{set}\left(d\tau'_{s_d}+\Delta T_s\right)$ takes the form

$$\Delta_{set}\left(d\tau'_{s_d}+\Delta T_s\right)=\sum_{r=1}^{d-1}M_rP\left([d-r+1]\tau'_{s_d}+\Delta T_s\right)+M_{d+1}P(\Delta T_s)+ M_d\left[P\left(\tau'_{s_d}+\Delta T_s\right)-1\right]=\sum_{r=1}^{d-1}M_rP'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)+ +M_{d+1}P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)+M_d\left[P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right]. \quad (30)$$

where $P'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)=P\left([d-r+1]\tau'_{s_d}+\Delta T_s\right)$.

To determine the condition when the following relation $\Delta_{\max}\left(d\tau'_{s_d}+\Delta T_{s.\max}\right)=\Delta_{\text{adm}}$ is true, let us find the conditions at which the expression (30) reaches its extremums. Here $\Delta T_{s.\max}=f\left(\tau''_{s_d},\Delta_{\text{adm}}\right)=\left\{\Delta t_{\text{in}_d}\right\}\cup\left\{\Delta t_{\text{out}_d}\right\}$, $\Delta T_{s.\max}\in\mathbb{R}$; $\tau''_{s_d}\in\tau'_{s_d}$; $\Delta T_{s.\max}$ determines the range of sampling time error relatively to τ''_{s_d}, at which channel symbol can be read without errors at given Δ, in the following way $d\tau''_{s_d}+\Delta T_s\in\left[-\left|\Delta t_{\text{in}_d}\right|+d\tau''_{s_d};d\tau''_{s_d}\right]\cup\left[d\tau''_{s_d};d\tau''_{s_d}+\Delta t_{\text{out}_d}\right]$.

First of all, we solve the following problem

$$\left.\begin{aligned}&\frac{\partial\Delta_{\text{set}}\left(d\tau'_{s_d}+\Delta T_{s.\max}\right)}{\partial M_1}=0\\&\frac{\partial\Delta_{\text{set}}\left(d\tau'_{s_d}+\Delta T_{s.\max}\right)}{\partial M_2}=0\\&\vdots\\&\frac{\partial\Delta_{\text{set}}\left(d\tau'_{s_d}+\Delta T_{s.\max}\right)}{\partial M_{d+1}}=0\end{aligned}\right\}\qquad(31)$$

After a series of simple transformations, system (31) will take the form

$$\left.\begin{aligned}&P'_{1,d+1}\left(\tau'_{s_d},\Delta T_{s.\max}\right)=0\\&P'_{2,d+1}\left(\tau'_{s_d},\Delta T_{s.\max}\right)=0\\&\vdots\\&P'_{d,d+1}\left(\tau'_{s_d},\Delta T_{s.\max}\right)=1\\&P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_{s.\max}\right)=0\end{aligned}\right\}\qquad(32)$$

In general case the solution (32) doesn't exist for the same reasons as for the system (22). It follows from the system (32) that the signal amplitude does not affect the reaching extremums, therefore extremums occur at the boundaries of the signal constellation.

Therefore, for sequence consisting of d-th symbols we need solve $2^{d+1}-1$ equations to obtain $\Delta T_{s.\max}$ due to the same number of amplitude combinations that which corresponds to the sub exponential time complexity of the algorithm.

To further simplify the computational complexity of the algorithm, consider the expression (30) rewriting it in the following way

$$s_{\text{out}}\left(d\tau'_{s_d}+\Delta T_s\right)=S_p\left(\tau'_{s_d},\Delta T_s\right)+S_n\left(\tau'_{s_d},\Delta T_s\right),\qquad(33)$$

where

$$S_p\left(\tau'_{s_d},\Delta T_s\right)=\sum_{k_1=1}^{d-1}M_{k_1}\left|P'_{k_1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\chi_+\left(P'_{k_1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+$$
$$+M_{d+1}\left|P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\chi_+\left(P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+$$
$$M_d\left|P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right|\chi_+\left(P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right)$$

$$S_n\left(\tau'_{s_d},\Delta T_s\right)=\sum_{k_2=1}^{d-1}M_{k_2}\left|P'_{k_2,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\chi_-\left(P'_{k_2,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+$$
$$+M_{d+1}\left|P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\chi_-\left(P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+$$
$$+M_d\left|P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right|\chi_-\left(P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right).$$

It is obvious that to determine the conditions when $\left|\Delta_{\text{set}}\left(d\tau'_{s_d}+\Delta T_s\right)\right|\to\max$, we need that one of the following conditions set come true:

1st set

$$\forall\tau'_{s_d},\forall d:S_p\left(\tau'_{s_d},\Delta T_s\right)\to\max,\qquad(34)$$

$$\forall\tau'_{s_d},\forall d:\left|S_n\left(\tau'_{s_d},\Delta T_s\right)\right|\to\min.\qquad(35)$$

2nd set

$$\forall\tau'_{s_d},\forall d:S_p\left(\tau'_{s_d},\Delta T_s\right)\to\min,\qquad(36)$$

$$\forall\tau'_{s_d},\forall d:\left|S_n\left(\tau'_{s_d},\Delta T_s\right)\right|\to\max.\qquad(37)$$

The solution for the first set is $\forall M_{k_1}=M_{\text{sc}_n}$, $k_1=\overline{1,d+1}$ (problem (34)) and $\forall M_{k_2}=M_{\text{sc}_1}$, $k_2=\overline{1,d+1}$ (problem (35)). For second set solution has the following form: $\forall M_{k_1}=M_{\text{sc}_1}$, $k_1=\overline{1,d+1}$ (problem (36)) and $\forall M_{k_2}=M_{\text{sc}_n}$, $k_2=\overline{1,d+1}$ (problem (37)).

Considering the above obtained results expression for $\Delta_{\max}\left(d\tau'_{s_d}+\Delta T_s\right)$ will take the form

$$\Delta_{\max}\left(d\tau'_{s_d}+\Delta T_s\right)=\left|S'_+\left(\tau'_{s_d},\Delta T_s\right)1\left(\left|S'_+\left(\tau'_{s_d},\Delta T_s\right)\right|-\left|S'_-\left(\tau'_{s_d},\Delta T_s\right)\right|\right)+\right.$$
$$\left.+S'_-\left(\tau'_{s_d},\Delta T_s\right)1\left(\left|S'_-\left(\tau'_{s_d},\Delta T_s\right)\right|-\left|S'_+\left(\tau'_{s_d},\Delta T_s\right)\right|\right)\right|\Phi'\left(\tau'_{s_d},\Delta T_s\right),\qquad(38)$$

where

$$\Phi'\left(\tau'_{s_d},\Delta T_s\right)=$$
$$=1-\frac{1\left(\left|S'_+\left(\tau'_{s_d},\Delta T_s\right)\right|-\left|S'_-\left(\tau'_{s_d},\Delta T_s\right)\right|\right)\times1\left(\left|S'_-\left(\tau'_{s_d},\Delta T_s\right)\right|-\left|S'_+\left(\tau'_{s_d},\Delta T_s\right)\right|\right)}{2};$$

$$S'_+\left(\tau'_{s_d},\Delta T_s\right)=\sum_{r=1}^{d-1}\left|{}'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\left[M_{\text{sc}_n}\chi_+\left(P'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+\right.$$
$$\left.+M_{\text{sc}_1}\chi_-\left(P'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)\right]+\left|P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\times$$
$$\times\left[M_{\text{sc}_n}\chi_+\left(P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+M_{\text{sc}_1}\chi_-\left(P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)\right]+$$
$$+\left|P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right|\left[M_{\text{sc}_n}\chi_+\left(P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right)+\right.$$
$$\left.M_{\text{sc}_1}\chi_-\left(P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right)\right];$$

$$S'_-\left(\tau'_{s_d},\Delta T_s\right)=\left|\sum_{r=1}^{d-1}\left|P'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\left[M_{sc_1}\chi_+\left(P'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+\right.\right.$$
$$\left.+M_{sc_n}\chi_-\left(P'_{r,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)\right]+\left|P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right|\times$$
$$\times\left[M_{sc_1}\chi_+\left(P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)+M_{sc_n}\chi_-\left(P'_{d+1,d+1}\left(\tau'_{s_d},\Delta T_s\right)\right)\right]+$$
$$+\left|P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right|\left[M_{sc_1}\chi_+\left(P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right)+\right.$$
$$\left.\left.+M_{sc_n}\chi_-\left(P'_{d,d+1}\left(\tau'_{s_d},\Delta T_s\right)-1\right)\right]\right|.$$

Using expression (38) the desired equation $\Delta_{\max}\left(d\tau''_{s_d}+\Delta T_{s.\max}\right)=\Delta_{\text{adm}}$ takes the form

$$\Delta_{\max}\left(d\tau''_{s_d}+\Delta T_{s.\max}\right)=\left|S'_+\left(\tau''_{s_d},\Delta T_{s.\max}\right)1\times\right.$$
$$\times\left(\left|S'_+\left(\tau''_{s_d},\Delta T_{s.\max}\right)\right|-\left|S'_-\left(\tau''_{s_d},\Delta T_{s.\max}\right)\right|\right)+$$
$$+S'_-\left(\tau''_{s_d},\Delta T_{s.\max}\right)1\left(\left|S'_-\left(\tau''_{s_d},\Delta T_{s.\max}\right)\right|-\left|S'_+\left(\tau''_{s_d},\Delta T_{s.\max}\right)\right|\right)\Big|\times \tag{39}$$
$$\times\Phi'\left(\tau''_{s_d},\Delta T_{s.\max}\right).$$

Analyzing (39) we can conclude that we have the polynomial time complexity of the algorithm to estimate $\Delta T_{s.\max}$, because we need to solve only one equation with 2(d+1) polynomials.

2.2 Algorithm for Resolution Time Estimation

In this section the algorithm for capacity, resolution time estimation is presented in the form a brief description of the main stages of its execution.

But first of all, it should be noted that by virtue of the validity of the transposition property for LTI system and the cyclo-stationary nature of the behavior of the PAM-n-signal on its output due ISI the following relation will be true

$$\lim_{d\to\infty} t''_d = t_{\text{res}}, \tag{40}$$

where $t''_d=\left\{\Delta t_{\text{in}_d}+\tau''_{s_d}\right\}\cup\left\{\tau''_{s_d}+\Delta t_{\text{out}_d}\right\}$.

From equation (40) the following relation follows

$$\left|t''_d-t_{\text{res}}\right|\le\varepsilon_{\text{res}}, \tag{41}$$

where ε_{res} is the precision with which the resolution time t_{res} is determined using the greatest settling time t''_d for the d-th symbol. The value of ε_{res} in fact, the largest value of the residual data dependent jitter level is determined by approximating the resolution time by the largest settling time for the d-th symbol.

Algorithm listing

1. According to the previous papers on the theory of resolution time [8-14], at first step we need to estimate the greatest settling time for the third symbol t'_3 using any numerical method for equation solution (28) with time accuracy ε_{res}.

2. The parameter estimation ε is made based on resolution time approximation accuracy ε_{res} in the following form

$$\varepsilon=\min\left\{\varepsilon'_+;\varepsilon'_-\right\},$$
$$\varepsilon'_+=\min_{T_+}\left|H\left(T_+\right)-H\left(T_++\varepsilon_{\text{res}}\right)\right|, \tag{42}$$
$$\varepsilon'_-=\min_{T_-}\left|H\left(T_-\right)-H\left(T_--\varepsilon_{\text{res}}\right)\right|,$$

where

$$T_+=\left\{t'_{\text{w.st.i,3}}\right\}_{i=1}^{p};\ T_-=\left\{t'_{\text{w.end.i,3}}\right\}_{i=1}^{p};$$
$$H\left(x\right)=\left|H_+\left(x\right)1\left(\left|H_+\left(x\right)\right|-\left|H_-\left(x\right)\right|\right)+\right.$$
$$\left.+H_-\left(x\right)1\left(\left|H_-\left(x\right)\right|-\left|H_+\left(x\right)\right|\right)\right|\zeta\left(x\right),$$

is maximum signal amplitude, providing the maximum settling error, at the output of channel after transmission information sequence, consisting of three symbols, is transmitted. The amplitudes of sequence should provide the maximum settling error.

$$H_+\left(x\right)=\sum_{r=1}^{2}\left|P\left(\left[4-r\right]x\right)\right|\left[M_{sc_n}\chi_+\left(P\left(\left[4-r\right]x\right)\right)+M_{sc_1}\chi_-\left(P\left(\left[4-r\right]x\right)\right)\right]+$$
$$+\left|P\left(x\right)\right|\left(M_{sc_n}\chi_+\left(P\left(x\right)-1\right)+M_{sc_1}\chi_-\left(P\left(x\right)-1\right)\right);$$
$$H_-\left(x\right)=\sum_{r=1}^{2}\left|P\left(\left[4-r\right]x\right)\right|\left[M_{sc_1}\chi_+\left(P\left(\left[4-r\right]x\right)\right)+M_{sc_n}\chi_-\left(P\left(\left[4-r\right]x\right)\right)\right]+$$
$$+\left|P\left(x\right)\right|\left(M_{sc_1}\chi_+\left(P\left(x\right)-1\right)+M_{sc_n}\chi_-\left(P\left(x\right)-1\right)\right)$$
$$\zeta\left(x\right)=1-\frac{1\left(\left|H_+\left(\tau_s\right)\right|-\left|H_-\left(\tau_s\right)\right|\right)\times1\left(\left|H_-\left(\tau_s\right)\right|-\left|H_+\left(\tau_s\right)\right|\right)}{2}$$

The number of symbols in information sequence equal to three is chosen taking into account the results of papers [8-18].

3. Parameters ($k_c=\dfrac{\hat{\sigma}_{cC}-\hat{\sigma}_{cD}}{C-D}$, $b_c=\dfrac{C\hat{\sigma}_{cD}-D\hat{\sigma}_{cC}}{C-D}$) estimations are calculated for three ($c=\overline{1;3}$) majorizing series (see table 1) utilizing the following rule $C,D\in H=\left\{h\,|\,h\in\mathbb{N}^*\right\}$, $C>D$ (it is advisable to choose $C=10$ и $D=3$) and expressions for $\hat{\sigma}_{\cdot H}$ (see table 1 and utilizing rules presented in table 2).

After estimation $\hat{\sigma}_{\cdot H}$ the majorized series are reconstructed utilizing $\sigma_{ch}=k_c h_c+b_c$. Where S_H – number of maxima $\left|P_{d-H+1,d}\left(.\right)\right|$ for given value H; $d>H$.

4. The choice of the most appropriate type c_{opt} of majorizing series is carried out on the basis of the analysis of the set $L_{\text{rem}}=\left\{h'_c\,|\,\forall q\in\left[h'_c;l'\right)\ \left(u_{cq}\left(T_{\text{com}}\right)-\left|P_{l'-H+1,l'}\left(T_{\text{com}}\right)\right|\ge0\right);c=\overline{1,3}\right\}$, which determines the numbers of terms used to estimate the residuals of the series. Here u_{cq} is q-th term of the functional c-th majorizing series; $T_{\text{com}}\in\left[t'_{\text{w.st.1,3}};\tilde{\tau}_s^{(3)}\right)$. Note that in the analysis it is advisable to limit the value $l'\le20$. If h'_c takes different values for each type of majorizing series, then the estimate c_{opt} will be in the form [14]

$$c_{\text{opt}}=\arg\min_{c=\overline{1;3}}\sum_{q=1}^{3}\delta_{qc}h'_q \tag{43}$$

Table 1

Majorizing series and some properties [14]

Parameter	1st type series (c=1)	2nd type series (c=2)	3rd type series (c=3)
Series	$\sum_{h=1}^{\infty} u_{1h}(x_s) = \sum_{h=1}^{\infty} \exp(-\sigma_{1h} x_s)$	$\sum_{h=1}^{\infty} u_{2h}(x_s) = \sum_{h=1}^{\infty} \sigma_{2h} x_s \exp(-\sigma_{2h} x_s)$	$\sum_{h=1}^{\infty} u_{3h}(x_s) = \sum_{h=1}^{\infty} (1+\sigma_{3h} x_s)\exp(-\sigma_{3h} x_s)$
$\hat{h}_\bullet$	$\hat{h}_1 = \left\lceil -\left(\frac{\ln\left(R_{\hat{h}_1}^{1} k_1 x_s\right)}{x_s k_1} + \frac{b_1}{k_1} + 1 \right) \right\rceil$	$\hat{h}_2 = \left\lceil -\left[\frac{1 + W_{-1}\left(R_{\hat{h}_2}^{2} k_2 x_s \exp(-2)\right)}{k_2 x_s} + \frac{b_2}{k_2} + 1 \right] \right\rceil$	$\hat{h}_3 = \left\lceil -\left[\frac{2 + W_{-1}\left(R_{\hat{h}_3}^{3} k_3 x_s \exp(-2)\right)}{k_3 x_s} + \frac{b_3}{k_3} + 1 \right] \right\rceil$
$\hat{\sigma}_{\bullet H}$	$\sigma_{1H} = -\frac{\ln E_H}{\tilde{\tau}_s^{(H)}}$	$\sigma_{2H} = -\frac{W_{-1}(-E_H)}{\tilde{\tau}_s^{(H)}}$	$\sigma_{3H} = -\frac{1 + W_{-1}\left[-E_H \exp(-1)\right]}{\tilde{\tau}_s^{(H)}}$
Comments: W_{-1} is Lambert W function with branch -1 selection, $\lceil . \rceil$ is ceil operation			

Table 2

Rules used in the calculation parameters k_c, b_c

Are damped oscillations observed during the settling of the impulse response?
No
$E_H = \left|P_{d-H+1,d}\left(\tilde{\tau}_s^{(H)}\right)\right| = \min_{\substack{\tau_s' \to \max \\ \tau_s' \in \left[t'_{\text{w.st.}1,3};3t'_{\text{w.end.}1,3}\right]}} \left\{ \left|P_{d-H+1,d}(\tau_s')\right| : \left|P_{d-H+1,d}(\tau_s')\right| > 0 \right\}$
Yes
$E_H = \left|P_{d-H+1,d}\left(\tilde{\tau}_s^{(H)}\right)\right|$ $\tilde{\tau}_s^{(H)} = \max T_H'$ $T_H' = \left\{ \tau_{\text{M}_{K_i}} \in \tau_s' = \left[t'_{\text{w.st.}1,3}, 3t'_{\text{w.end.}p,3}\right] \mid \forall \tau_s' \in \overset{\circ}{U}\left(\tau_{\text{M}_{K_i}}\right) \times \right.$ $\left. \times \left(\left|P_{d-H+1,d}(\tau_s')\right| \le \left|P_{d-H+1,d}\left(\tau_{\text{M}_{K_i}}\right)\right| \right) ; K_i = \overline{1, S_H} \right\}$

If h_c' takes the same values for at least two types of majorizing series, then the estimation c_{opt} is made according to the rule

$$c_{\text{opt}} = \arg\min_{c \in I'} \times$$
$$\times \left(\max_{T_{\text{com}}} \left\{ u_{ch_c'}(T_{\text{com}}) - P_{l'-h_c'+1,l'}(T_{\text{com}}) \middle| u_{ch_c'}(T_{\text{com}}) - P_{l'-h_c'+1,l'}(T_{\text{com}}) > 0 \right\} \right) \tag{44}$$

where $I' = \arg\min_{c=\overline{1;3}} \sum_{q=1}^{3} \delta_{qc} h_q'$.

5. The dependence of the effective channel memory on the symbol duration is estimated as follows:

$$\hat{G}(\tau_s) = \min\left\{ G'(\tau_s) : 0 < R_{\hat{h}_{c_{\text{opt}}}}^{c_{\text{opt}}} + \sum_{r_1=1}^{\hat{h}_{c_{\text{opt}}}} \left|P(r_1\tau_s)\right| - \sum_{r_2=1}^{G'(\tau_s)+1} \left|P(r_2\tau_s)\right| \le \varepsilon \right\},$$

where $R_{\hat{h}_{c_{\text{опт}}}}^{c_{\text{опт}}} = Q_\varepsilon \varepsilon$, $Q_\varepsilon \in [0{,}1; 0{,}01]$; $\hat{h}_{c_{\text{опт}}}$ determined on the basis $R_{\hat{h}_{c_{\text{опт}}}}^{c_{\text{опт}}}$ utilizing $\hat{h}_\bullet$, presented in table 2.

Next, the effective memory is evaluated:

$$G = \begin{cases} \hat{G}\left[\min(t_3')\right] - 1, & \text{if } \hat{G}\left[\min(t_3')\right] - 1 \ge 3, \\ 2, & \text{if } \hat{G}\left[\min(t_3')\right] - 1 < 3. \end{cases} \tag{45}$$

Then capacity estimation is made according (16) taking into account $t_{\text{res}} = t''_{G+1}$.

Conclusion

A novel method for capacity and resolution time estimations with polynomial computational complexity that does not depend on the size of the channel alphabet of PAM-n-signals and is determined only by the effective memory value. The shape of pulse of PAM-n-signals has arbitrary form and amplitudes take only positive values.

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ТЕОРИЯ РАЗРЕШАЮЩЕГО ВРЕМЕНИ В ОБЛАСТИ СИСТЕМ ШИРОКОПОЛОСНОГО ДОСТУПА. АЛГОРИТМ ОЦЕНКИ ДЖИТТЕРА, ОБУСЛОВЛЕННОГО ПЕРЕДАЧЕЙ ДАННЫХ, И ПРОПУСКНОЙ СПОСОБНОСТИ С ПОЛИНОМИАЛЬНЫМ ВРЕМЕНЕМ ИСПОЛНЕНИЯ

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Аннотация

В настоящее время в связи с постоянным ростом объема передаваемой информации все большее внимание уделяется вопросам возможности увеличения скорости передачи за счет использования режима передачи "выше скорости Найквиста". Несмотря на актуальность данной темы исследования, из-за достаточно больших математических трудностей получить значимые результаты затруднительно. Одной из теорий, позволяющих совершить прорыв в данной области, является бурно развивающаяся теория времени разрешения, разработанная для фазовых радиотехнических систем передачи данных и информационно-измерительных оптико-электронных систем с АИМ-сигналами прямоугольной формы. В данной статье представлен новый метод оценки пропускной способности и разрешающего времени с полиномиальной вычислительной сложностью, которая не зависит от размера канального алфавита АИМ-n-сигнала и определяется только объемом эффективной памяти канала, при этом форма импульса является произвольной, а амплитуды принимают только положительные значения. Метод также позволяет оценить время, ограничивающее возможные отклонения из-за ошибки выборки во времени, при котором информация о канальном символе будет считана безошибочно или с заданной вероятностью ошибки.

Ключевые слова: *МСИ, разрешающее время, теория разрешающего времени, АИМ-сигналы, пропускная способность, джиттер, зависящий от данных.*

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