USING TWO-DIMENSIONAL FAST FOURIER TRANSFORM FOR ESTIMATING SPECTRAL CORRELATION FUNCTION

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The paper presents the algorithm for estimating spectral correlation function (SCF) of a wide-sense cyclostationary random process. SCF provides the quantitative representation of the correlation in frequency domain and relates to cyclic autocorrelation function via Fourier transform. The algorithm is based on two-dimensional Fourier transform, which is being applied to the discrete diadic correlation function weighted by a two-dimensional windowing function, chosen rectangular in the direction orthogonal to the current-time axis. This transform can be implemented by means of the fast Fourier transform (FFT) algorithm, which is built-in in a variety of modern mathematical platforms. A pulse-amplitude modulated process masked by the additive stationary Gaussian noise was considered as an example of a random process exhibiting strong cyclostationarity. The numerical simulation where the estimation of spectral correlation function of such process is conducted, and it proved the effectiveness of the proposed algorithm.

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1. Introduction

Signals used in engineering have a typical common property which is their structural repeatability. This property underlies the processes of their formation. Signals of animate and inanimate nature also have a repeatability. Such signals are characterized by "latent periodicity". The combination of randomness and periodicity is the property of "cyclostationarity".

Nowadays one of the major tasks to be overcome by means of signal processing techniques based on the models representing signals as realizations of cyclostationary random processes [1, 2] is the non-parametric estimation of the cyclic characteristics of a random process involving long digital samples.

There are some algorithms to implement cyclic spectral estimation [3]. The earliest and most popular, which have a relatively high performance, are Fast Fourier Transform Accumulation Method (FAM) [4], which is based on Wigner-Ville sample transformation, and Spectral Strip Correlation analyzer (SSCA) [5]. But the estimators based on them can't guarantee the absence of missing components of the cyclic spectra because they do not cover the bispectral plane completely.

Over the past decades the significant progress in computer capacity has happened. It has allowed the creation of new methods for cyclic spectral estimation which do not skip any components of spectral correlation functions during the analysis. The algorithms based on cyclic periodograms, or cyclograms, are of interest: the double-length FFT estimator (2N-FFT) [6] and the averaged absolute spectral correlation density (AASCD) estimator [7]. But they have a drawback which is the large requirements of computational resources, the performance of central processor unit (CPU) and especially the capacity of random-access memory (RAM).

The problem of finding a simple but computationally efficient algorithm for estimation of spectral correlation function of random processes remains relevant. The algorithm must be easy to implement, and the standard well-known building blocks should be used as much as possible. Such an algorithm that could be a conceptual analogue of the well-known Fast Fourier Transform (FFT), used for the classical non-parametric spectral analysis [8], could be very profitable for both scientific and engineering usages.

The paper presents the algorithm for spectral correlation estimation for the finite-length observations of random processes based on two-dimensional fast Fourier transform. This technique intensively used in image processing [9] rather than in the classical signal processing. The main idea is the application of this transform to the windowed correlation matrix of the observed signal, where the purpose of the windowing is to increase the robustness.

The rest of the chapter is organized as follows. Section 2 introduces the main cyclostationary characteristics used for description of non-stationary random signals. The proposed algorithm for the estimation of the spectral correlation function are described in Section 3. Section 4 presents the results of numerical simulation, which are compared with analytic solution. The chapter ends with the conclusion.

2. Cyclostationary characteristics

The dyadic autocorrelation function depending on two time instants for a wide-sense cyclostationary random process $x(t)$ with zero mean $\mathbb{E}\{x(t)\}=0$ can be described as:

$$R_s(t_1,t_2)=\mathbb{E}\left\{x(t_1)x^*(t_2)\right\}, \tag{1}$$

where $\mathbb{E}\{\bullet\}$ designates the probabilistic expectation and the superscript $*$ designates the complex conjugation.

More convenient in some cases is the two-dimensional correlation function which is formed by changing the variables t_1, t_2 with:

$$\begin{cases} t=\dfrac{t_1+t_2}{2}, \\ \tau=t_1-t_2. \end{cases} \tag{2}$$

After replacement the following symmetric form of the two-dimensional autocorrelation function (2D-ACF) is:

$$\mathcal{R}_s(t,\tau)=R_s\left(t+\tau/2,t-\tau/2\right), \tag{3}$$

where τ means the relative time, or the time shift between two instants, where the correlation is evaluated, t means the current time. Expression (3) can be expanded into the generalized Fourier series of the current time t:

$$\mathcal{R}_x(t,\tau)=\sum_{\alpha\in \mathrm{A}_2}\mathcal{R}_x^\alpha(\tau)\exp(j2\pi\alpha t), \tag{4}$$

where A_2 is a countable set, the elements α of which are called cyclic frequencies, and the coefficients of the series $\mathcal{R}_x^\alpha(\tau)$ are functions of the argument τ. A countable set of functions $\left\{\mathcal{R}_x^\alpha(\tau)\right\}$, composed of nonzero coefficients, forms the cyclic autocorrelation function (CACF). In turn, the coefficients $\mathcal{R}_x^\alpha(\tau)$ make up the components, or sections, of the 2D-ACF. CACF is equivalent to the 2D-ACF in the same sense as the set of Fourier series coefficients could replace the periodic signal.

Consider the component $\mathcal{R}_x^0(\tau)$ at $\alpha=0$. One can notice, that it always exists and it corresponds to the one-dimensional time-invariant autocorrelation function that can fully characterize a wide-sense stationary zero-mean random process:

$$\forall t:\mathcal{R}_s(t,\tau)=\mathcal{R}_x^0(\tau). \tag{5}$$

If the suggestion (5) does not hold, it will mean that the random process $x(t)$ is non-stationary. This automatically makes the problem of its formal investigation extremely difficult unless the expansion (4) remains valid leading to the cyclostationary case. The above reasoning show that stationary in a wide-sense random process can be considered a particular case of cyclostationary in a wide-sense random process.

In case of a strictly periodic behavior of 2D-ACF (3), when it reproduces exactly with the period T:

$$\mathcal{R}_\xi(t,\tau)=\mathcal{R}_\xi(t+T,\tau), \tag{6}$$

the coefficients of its Fourier series can be numbered using an integer index k:

$$\mathrm{A}_2=\left\{\alpha \mid \alpha=\frac{k}{T},\quad k\in\mathbb{Z}\right\}, \tag{7}$$

where $1/T$ is the fundamental frequency, which is reciprocal to the period T.

Then Fourier series (4) can be presented in the form:

$$\mathcal{R}_x(t,\tau)=\sum_{k=-\infty}^{+\infty}\mathcal{R}_x^{k/T}(\tau)\exp\left(j2\pi\frac{k}{T}t\right). \quad (8)$$

To determine the component $\mathcal{R}_x^{k/T}(\tau)$, it will be enough to take the Fourier transform by integrating the 2D-ACF over one period of its repetition [10]:

$$\mathcal{R}_x^{k/T}(\tau)=\frac{1}{T}\int_0^T\mathcal{R}_x(t,\tau)\exp\left(-j2\pi\frac{k}{T}t\right)dt. \quad (9)$$

Fourier transform of each component $\mathcal{R}_x^{\alpha}(\tau)$ gives spectral components:

$$\mathcal{S}_x^{(\alpha)}(f)=\int_{-\infty}^{+\infty}\mathcal{R}_x^{(\alpha)}(\tau)\exp(-j2\pi f\tau)d\tau. \quad (10)$$

The set of such components $\left\{\mathcal{S}_\xi^{(\alpha)}(f)\right\}$ build up the spectral correlation function (SCF) of the random process, which provides the correlation description in frequency domain.

SCF can be expressed as a function of two continuous arguments by means of generalized Dirac delta functions:

$$\mathcal{S}_x(\alpha,f)=\sum_{\nu\in\mathrm{A}}\mathcal{S}_x^{(\nu)}(f)\delta(\alpha-\nu), \quad (11)$$

where the auxiliary ν is used as a bound element in the summation. Such a function of two real variables has the meaning of spectral correlation density (SCD), which is used instead of SCF when one deals with finite-time observations of random processes in practice. SCD is a function of continuous cyclic frequency rather than discrete.

If one applies the similar replacement for frequency variables as in time domain (2) for ACF, namely:

$$\begin{cases} f=\dfrac{f_1+f_2}{2}, \\ \alpha=f_1-f_2, \end{cases} \quad (12)$$

then SCD can be rewritten in the functional form depending on two frequencies f_1 and f_2:

$$S_x(f_1,f_2)=\mathcal{S}_x\left(f_1-f_2,\frac{f_1+f_2}{2}\right). \quad (13)$$

This bifrequency spectral representation of correlation (13) can be evaluated as the probabilistic expectation:

$$S_x(f_1,f_2)=\mathbb{E}\left\{X(f_1)X^*(f_2)\right\}, \quad (14)$$

where $X(f)$ is a random complex measure [11] that can be considered as the spectral density of the random process $x(t)$ via Crámer's representation [12]:

$$x(t)=\int_{-\infty}^{+\infty}\exp(j2\pi ft)X(f)df. \quad (15)$$

Having taken the two-dimensional Fourier transform to ACF $R_x(t_1, t_2)$ one obtains the density $S_x(f_1, f_2)$:

$$S_x(f_1,f_2)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}R_x(t_1,t_2)\exp\left[-j2\pi(f_1t_1-f_2t_2)\right]dt_2dt_1, \quad (16)$$

where the integral is considered in the generalized or distributional sense [13] that leads to an arbitrary mixture of regular and generalized functions.

The possible change of the ACF for its estimation in (16) opens the road to the design of relatively simple and fast algorithm for the estimation of the SCF (10).

3. Spectral Correlation Function Estimation

Consider the realization $x(t)$ of the continuous-time random process observed over the finite-time interval $[0, T_x)$. Suppose that the length of the observation is much greater than the maximal correlation time to be seen in the 2D-ACF (3) of the process: $T_x \gg \tau_{cor}$.

Then using uniform sampling of the realization $x(t)$ acquire digital signal of N samples $x[n] = x(nT_s)$, where T_s is the sampling period. The estimation of ACF in discrete time as two-dimensional sample function evaluates as follows:

$$\hat{R}_x[n_1,n_2]=x[n_1]x^*[n_2], \quad (17)$$

which is a function of two time instants

$$\hat{R}_x[n_1,n_2]=\hat{R}_x(n_1T_s,n_2T_s) \quad (18)$$

defined on the compact support $(t_1, t_2)\in[0, T_x]\times[0, T_x]$.

The estimator $\hat{R}_x[n_1,n_2]$ itself is not a consistent estimator for the true SCF (10) as one can see in [13]. To obtain a consistent estimator before using the transformation (16) an appropriate two-dimensional windowing has to be applied to (18).

There are different types of windows, lets focus on the simplest one:

$$w[n_1,n_2]=\mathrm{rect}\left(\frac{n_1-n_2}{d}\right)=\mathrm{rect}\left(\frac{t_1-t_2}{\Delta_w}\right)=w(t_1,t_2). \quad (19)$$

This window is rectangular one, where $d = \Delta_w/T_s$, and Δ_w defines the width of the window in the direction parallel to the line $t_1 = -t_2$; the function rect(ν) is defined as follows:

$$\mathrm{rect}(\nu)=\begin{cases}1, |\nu|<1/2, \\ 0, |\nu|>1/2.\end{cases} \quad (20)$$

There are some restrictions on the choice of window length. Firstly, the choice of excessively wide window inevitably leads to the higher variation of the estimated SCD (16). Secondly, the width Δ_w of the windows $w(t_1, t_2)$ does not have to be chosen less than twice as short as the expected length of the maximal correlation time of the processes τ_{cor}: $\Delta_w \ge 2\tau_{cor}$.

Applying the window (19) to ACF (18) yields:

$$\tilde{R}_x[n_1,n_2]=\hat{R}_x[n_1,n_2]w[n_1,n_2], \quad (21)$$

which is to be transformed by the two-dimensional discrete Fourier transform (DFT):

$$\tilde{S}_x[m_1,m_2]=\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}\tilde{R}_x[n_1,n_2]\exp\left[-j\frac{2\pi}{N}(n_1m_1-n_2m_2)\right]. \quad (22)$$

This transformation can be realized by very popular and well-known fast Fourier transform (FFT) algorithm, which is available on majority modern mathematical platforms.

The estimation of SCD $S_x(f_1, f_2)$ defined by (22) is equal $\tilde{S}_x[m_1, m_2]$ multiplied by the factor T_s^2:

$$\hat{S}(f_1, f_2) = T_s^2 \tilde{S}_x[m_1, m_2]. \tag{23}$$

In order to move from bifrequency representation of SCD (13) to representation (11), where one of the arguments is cyclic frequency, one need to do the following:

$$\hat{\mathcal{S}}_x(\alpha, f) = \hat{S}_x(f + \alpha/2, f - \alpha/2). \tag{24}$$

Actually, the particular value of the cyclic frequency α defines the straight line in the bifrequency plane (f_1, f_2), according to the equation $f_1 = f_2 + \alpha$, whereas the frequency axis f is defined alongside this line: $f = (f_1 + f_2)/2$.

To obtain the estimation of the SCF (10) from SCD (11) it is necessary to use scaling factor as follows:

$$\hat{S}_x^{\alpha}(f) = \frac{1}{T_x} \hat{\mathcal{S}}_x(\alpha, f). \tag{25}$$

As you can see from (25) the units of measurement for spectral correlation function and spectral correlation density differs. So, if the process under investigation is measured in volts, SCF will be measured in [V^2s] and SCD will be measured in [V^2s^2].

4. Numerical Simulation

Let us demonstrate the performance of the proposed algorithm by the numerical simulation, where the estimation of SCF is carried out. As example of random process [14] choose the process with strong cyclostationary properties. The regular periodic pulse train, where each pulse has the same waveform, but the amplitudes are random, is suitable for such description. This signal is also known as pulse-amplitude modulated (PAM):

$$x(t) = \sum_{q=-\infty}^{+\infty} A b_q \,\mathrm{rect}\left(\frac{t - qT}{\Delta}\right), \tag{26}$$

where A is the amplitude, Δ is the width of the pulse, T is the period, b_q are independent identically distributed (*i.i.d.*) Rademacher random variables with zero mean $\mathbb{E}\{A_q\} = 0$ and a finite variance:

$$\mathbb{E}\{b_q b_p\} = \begin{cases} 1, & p = q; \\ 0, & p \neq q. \end{cases} \tag{27}$$

Figure 1 demonstrates the typical realization of the PAM (26).

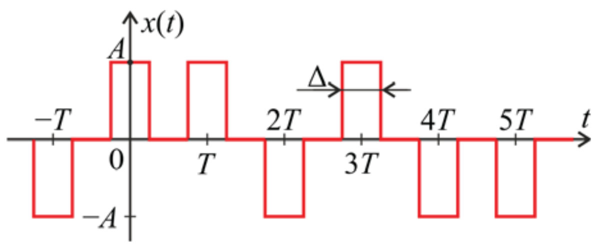

Fig. 1. A typical realization of the PAM random process

The rectangular form of each pulse in (26) provides the closed-form analytical expression [14]:

$$S_x^{\alpha}(f) = \frac{A^2 \Delta^2}{T} \mathrm{sinc}\left[\pi\left(f + \frac{\alpha}{2}\right)\Delta\right] \mathrm{sinc}\left[\pi\left(f - \frac{\alpha}{2}\right)\Delta\right], \tag{28}$$

where $\mathrm{sinc}(v) = \sin(v)/v$, complemented to a continuous function by setting 1 at $v = 0$.

The expression (28) can be explicitly used for further comparison.

The following simulation parameters were used for the parameters of the random process (26): the amplitude $A = 1$ V, the width of the pulse $\Delta = 5$ μs, the period $T = 2\tau$; the sampling period $T_s = \tau/16$, the number of samples $N = 4096$, the observation time $T_x = NT_s$. Thus, there are $T_x/T = 128$ totally observed within the observation time T_x. The width Δ_w of the window (19) is chosen a bit more than double the correlation time: $\Delta_w = 2.1\tau_{cor}$, since the one-side correlation time τ_{cor} is equal to the pulse width Δ for this kind of processes. A stationary Gaussian noise $z[n]$ with the uniform power spectral density (PSD) was added, so the signal-to-noise ratio (SNR) is as low as 0 dB.

Figure 2 demonstrates the intensity two-dimensional diagram of the absolute value of the estimated spectral correlation density, which gives one the full information about correlation in frequency domain.

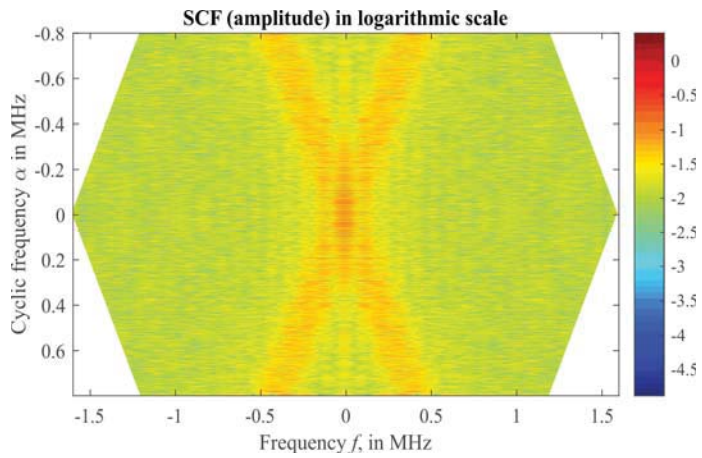

Fig. 2. The color intensity plot for the SCF estimation of the PAM process

It is difficult to determine the values of the cyclic frequencies characterizing the random process from the plot in fig. 2. The use of the integral characteristic [7]:

$$I(\alpha) = \int_{-F_{\max}}^{F_{\max}} \left|\mathcal{S}_x(\alpha, f)\right| df, \tag{29}$$

allows one to solve the above problem, where $F_{\max}$ is a frequency determined by the borders of the principal domain [15]:

$$F_{\max} = \frac{1}{2}\left(\frac{1}{T_s} - |\alpha|\right). \tag{30}$$

This characteristic, showed in fig.3, allows one to estimate the pseudo-power concentrating on each cyclic frequency.

Examining the figure 3 one can conclude that the strong cyclostationary of the process under investigation presents at cyclic frequencies that are multiples of $1/T = 0.1$ MHz.

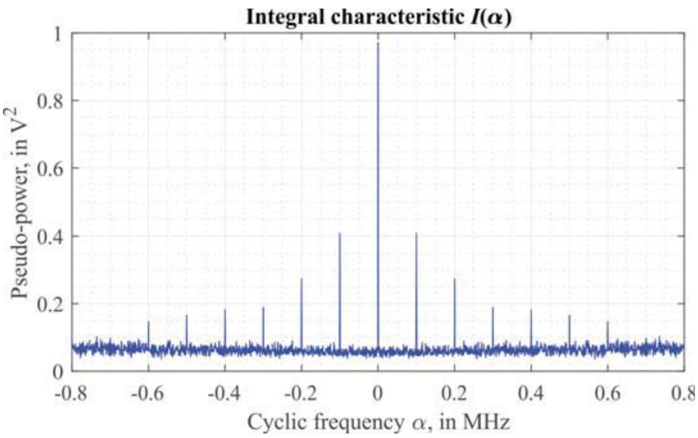

Fig. 3. The integral characteristic (pseudo-power) at cycle frequencies

Figures 4-7 shows the analytic (25) and experimental curves of the absolute value of several initial components of the SCF.

Figure 4 presents the SCF at $\alpha = 0$. It is the power spectrum density of the random process under investigation. The presence of additive stationary noise with the constant PSD leads to the location of estimated curve under analytical one.

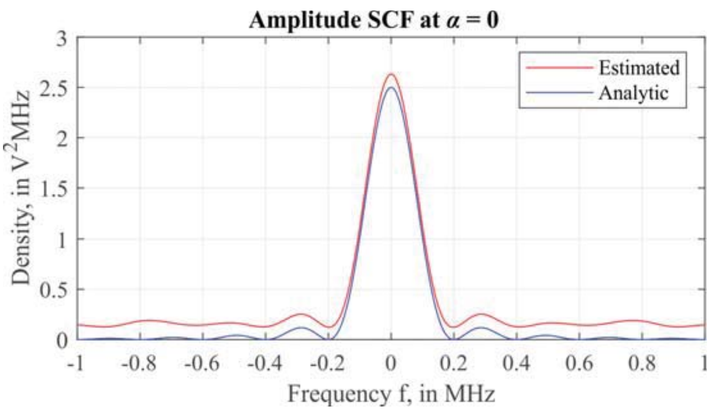

Fig. 4. The SCF component at zero cyclic frequency

Figure 5 shows the component of SCF at $\alpha = 1/T$. The curves almost coincide because of absence of cyclostationarity in the additive noise.

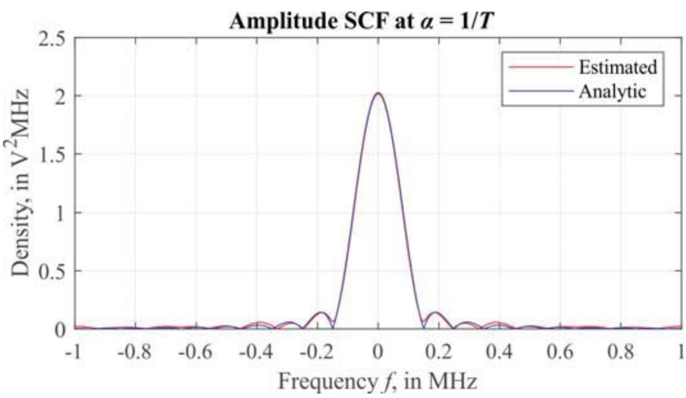

Fig. 5. The SCF component at $\alpha = 1/T$

Figure 6 demonstrates the second component of SCF at $\alpha = 2/T$. The difference between the estimated and analytic curves becomes slightly larger.

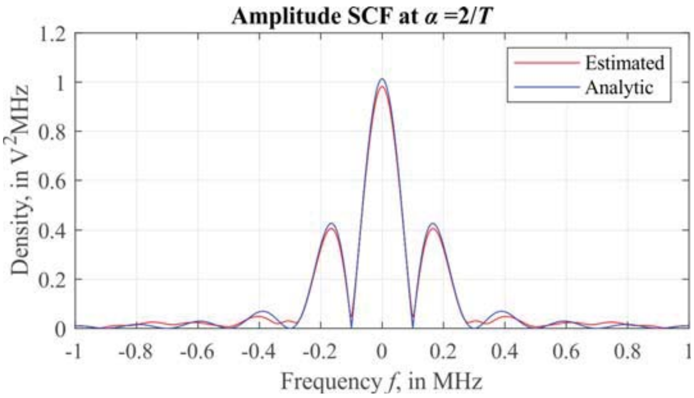

Fig. 6. The SCF component at $\alpha = 2/T$

Figure 7 shows the third component of SCF at $\alpha = 3/T$. The difference is even greater, especially at frequencies over than 0.5 MHz, where the values of the component itself is relatively small.

So, one can conclude, that the estimation error tends to increase as the component number becomes greater.

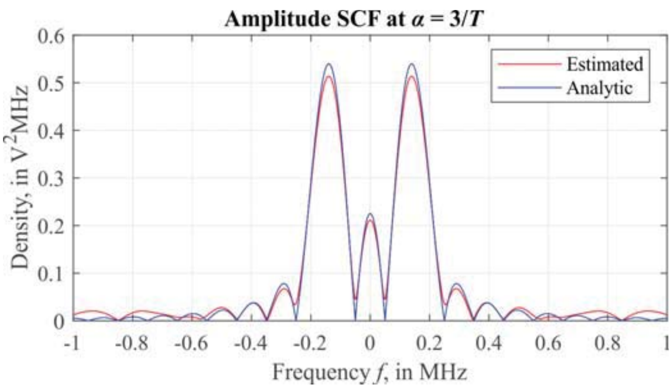

Fig. 7. The SCF component at $\alpha = 3/T$

5. Conclusion

The algorithm, proposed in this paper, allows one to estimate effectively the spectral correlation function of wide-sense cyclostationary processes. It reveals the cyclic frequencies exhibited by the process in case of a long finite-time observation of the process is available. This algorithm allows to avoid missing any SCF components by covering of the bifrequency plane that is dense enough.

The comparison, made between the estimated components of the SCF of the process with strong cyclostationary behavior, obtained by numerical simulation, and the curves drawn for their analytical expressions, has proved the effectiveness of the algorithm.

With an increase in component number of SCF, starting from the first one, the difference between the analytic and estimated curves becomes larger. It can be explained by the more sensitive of higher components to the influence of the noise.

To make the estimation of the spectral correlation function sufficient it is necessary to apply the window function, which is reduce the variance of the noise. The rectangular window allows to obtain the consistent estimate of SCF, but the further task for investigation can be the formal search for the best shape of the window and the optimal window width for the bias-variance trade-off.

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ИСПОЛЬЗОВАНИЕ ДВУМЕРНОГО ПРЕОБРАЗОВАНИЯ ФУРЬЕ ДЛЯ ОЦЕНКИ СПЕКТРАЛЬНОЙ КОРРЕЛЯЦИОННОЙ ФУНКЦИИ

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Аннотация
В настоящей работе представлен алгоритм оценивания спектральной корреляционной функции (СКФ) на основе конечных по длительности реализаций случайного процесса. СКФ является двухчастотным описанием вероятностных свойств циклостационарного в широком смысле случайного процесса и связана двойным преобразованием Фурье с его двумерной корреляционной функцией. В предложенном алгоритме двумерное дискретное преобразование Фурье (ДПФ) применяется к отсчётам дискретной двумерной корреляционной функции, взвешенной двумерной оконной функцией. Для уменьшения дисперсии шума оконная функция выбрана функция, имеющая прямоугольный профиль в направлении, перпендикулярном текущему времени. Двумерное ДПФ может быть реализовано с помощью алгоритма быстрого преобразования Фурье, реализованным в стандартных библиотеках большинства современных математических пакетов. Работа предложенного алгоритма продемонстрирована на примере оценивания СКФ аддитивной смеси последовательности импульсов со случайными амплитудами и стационарного гауссовского шума. Преимуществом предложенного в настоящей работе алгоритма является то, что за счёт достаточно плотного перекрытия двухчастотной плоскости он позволяет избежать пропуска компонент СКФ. Алгоритм, построенный на основе предложенного метода, представляет собой простой и эффективный инструмент анализа длинной реализации цифрового сигнала для выявления циклических свойств за счёт детального анализа в широкой полосе циклических частот.

Ключевые слова: *циклостационарность, циклическая частота, спектральная корреляционная функция, спектральная корреляционная плотность, двумерное БПФ, быстрое преобразование Фурье, псевдо-мощность.*

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