THE ALGORITHM FOR COHERENT PROCESSING OF WIDEBAND NON-BINARY SIGNAL-CODE STRUCTURES FOR SPEECH TRANSMISSION IN A DECAMETER RADIO CHANNEL

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This article presents the results of developing an algorithm for coherent processing of wideband non-binary orthogonal signal-code structures for voice transmission in a decameter radio channel. The structure of a radiogram, including preamble and data symbols, is presented, taking into account message length limitations for real-time voice transmission. An ensemble of orthogonal wideband phase-shift keyed signals is used as signals. The noise-correcting code used is a non-binary low-density parity-check code whose Galois field dimension is matched to the number of signals in the ensemble. Decoding is performed using a belief propagation algorithm, which involves calculating aposteriori probabilities for each possible transmitted symbol, taking into account observations. The article presents analytical expressions for calculating these probabilities, taking into account multipath signal propagation, assuming separation of the paths without their mutual interference, and also taking into account different a priori uncertainties regarding the complex channel transmission coefficients for each path. Three types of a priori information are assumed: fully known complex channel gains, an unknown phase shift of the channel gains, or an unknown phase shift and level of the channel gains. A fourth, widely used, quadratic ray summation option is also considered. In coherent processing, complex channel gains are assumed to be known or measured in some way. This article considers several options for measuring complex channel gains: using a separate measurement channel using the maximum likelihood method and using a Kalman filter. In one version, the Kalman filter operates on data from a separate measurement channel, while in another version, it operates on preamble symbols and on radiogram data symbols with feedback on decisions regarding the data symbols. Using the Waterson model of the ionospheric channel, this article substantiates the autoregressive order of the Kalman filter from the perspective of algorithm complexity and achievable noise immunity. Noise immunity curves are presented for the coherent processing algorithm using the Kalman filter and non-coherent processing algorithms. The corresponding energy gain is estimated.

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Introduction

In the contemporary world high frequency (HF) communication is widely used for organizing radio links in remote and hard-to-reach regions around the world. Apart from that, HF communication is applied for regions that have seriously suffered from destruction caused by natural disasters. The high reliability and cost-effectiveness make HF communication the optimal solution in both cases described above. The second area where HF radio links have found the application is communication in the Arctic and Antarctic. For instance, the Rosatom State Corporation has an interest in restoring the HF communication network to establish separate communication channels with the naval fleet of nuclear marine propulsions and land-based subscribers (see, for example, media reports [1]). The transmitted information consists of digital data and voice in real time.

One of the main drawbacks of the conventional HF communication is the availability of transmitted information, due to the opportunity to receive radio signals reflected from the ionosphere over vast areas of the globe. Consequently, challenges in ensuring the confidentiality of the transmitted information can be observed. The most reliable way to provide confidentiality at the physical level, i.e., without the use of cryptographic protection pf information, is to employ complex spread-spectrum modulation techniques to generate noise-like signals, and the use of special algorithms for processing such signals [1-4].

Earlier in 2020, MTUCI staff, including the authors of this article, conducted research in which wideband non-binary signal-code structures for real-time speech transmission were proposed [5]. However, the signal processing algorithm used in the work mentioned above assumes quasi-optimal non-coherent processing with quadratic summation of multipath components and the use of average estimation of channel coefficients instead of their true values [6-8]. Clearly, as shown in [9-13] for Rayleigh fading channels and in particular for narrowband HF channels [14-16, 28, 29] the noise immunity of digital radio link can be improved by adopting an optimal coherent processing algorithm.

The aim of the article is to develop an algorithm for coherent processing of wideband non-binary signal-code structures for speech transmission in a HF radio channel. Based on the methods proposed in [17-19] for joint optimal filtering of many channel parameters under conditions of a priori uncertainty and the algorithms presented in [20-22] for an ionospheric channel dispersion characteristic slope optimal filtering, the developed algorithm can be improved by jointly taking into account the results of optimal filtering of the channel coefficients and the slope of the dispersion characteristic.

Observation model for receiving one symbol of a radiogram

In this article we assume that multipath components can be separated without influencing each other (ideal separation of the beams). In this case, the output of the correlation receiver for each processed multipath component can be written as a vector of statistics

$$\dot{y}_{i,j} = E_s \dot{h}_j \delta_{ik} + \dot{n}_{i,j}, \quad (1)$$

where $\dot{y}_{i,j}$ is the signal at the output of the i-th correlator, processing the j-th multipath component while the symbol with the index k ($k = 1,...,2^m$) is being received, m is a bit depth of non-binary symbols, $\dot{h}_j$ is the channel gain for the j-th multipath component, E_s is energy of the received symbol, δ_{ik} is a Kronecker symbol, $\dot{n}_{i,j}$ is the noise part of statics. It is assumed that all transmitted symbols have the same energy level E_s.

The channel gains $\dot{h}_j$ are independent complex Gaussian variables for different multipath components and, in general, they are dependent in accordance with the Waterson model within one multipath component (diversity branch) for adjacent symbols in time [23].

The values $\dot{n}_{i,j}$ are complex Gaussian variables with zero mean and variance of the real and imaginary parts equal to σ_u^2.

The variance σ_u^2 is

$$\sigma_u^2 = E_s \sigma_n^2 = \frac{E_s N_0}{2}, \quad (2)$$

where σ_n^2 is the variance of the noise at the output of the correlator, $\frac{N_0}{2}$ is the level of the two-sided spectral power density of quasi-white noise in the main frequency range.

The instantaneous signal-to-noise ratio (SNR) for one multipath component is

$$SNR_j = \left|\dot{h}_j\right|^2 \frac{E_s}{N_0}. \quad (3)$$

The SNR for one multipath component, taking into account averaging over Rayleigh fading realizations, is

$$SNR_{mean,j} = 2\sigma_{A,j}^2 \frac{E_s}{N_0}. \quad (4)$$

where $\sigma_{A,j}^2$ is a parameter of the Rayleigh distribution of $\left|\dot{h}_j\right|$ for the j-th multipath component.

The total maximum SNR for optimal coherent reception of all multipath components [24] is

$$SNR_{mrc} = \frac{E_s}{N_0} \sum_{j=1}^{N_d} \left|\dot{h}_j\right|^2, \quad (5)$$

where N_d is number of multipath components (number of diversity branches).

The total average SNR is

$$SNR_{mean} = \left\langle \frac{E_s}{N_0} \right\rangle = \frac{E_s}{N_0} \sum_{j=1}^{N_d} \left(2\sigma_{A,j}^2\right). \quad (6)$$

The average SNR per bit taking into account the redundancy of the error-correcting code (coding rate r) and the symbol bit depth m:

$$\frac{E_b}{N_0} = \left\langle \frac{E_s}{N_0} \right\rangle \frac{1}{r \cdot m}. \quad (7)$$

General expressions for calculating posterior probabilities

First, the simple case of single-path propagation is considered. Then we extend it to the multipath case. The most complete statistics for making a decision on each symbol for each position of the received code block of a non-binary code is a set of posterior probabilities. So, in order to process the block, N vectors of 2^m posterior probabilities are demanded

$$P\left(c_k / \mathbf{y}\right),\ k=1,\ldots,2^m, \tag{8}$$

where $\mathbf{y}=\left[\dot{y}_1,\dot{y}_2,\ldots,\dot{y}_{2^m}\right]^T$ is a vector of 2^m outputs of a correlator processing a signal in a radiogram during the reception interval of one symbol out of N, c_k is a proposed received symbol. Decoding a code block of a non-binary LDPC code using the belief propagation algorithm requires computing the mentioned probabilities.

According to the product rule of probabilities

$$dP(\mathbf{y},c_k)=W\left(\mathbf{y},c_k\right)d\mathbf{y}=P\left(c_k / \mathbf{y}\right)W\left(\mathbf{y}\right)d\mathbf{y}, \tag{9}$$

where $dP(\mathbf{y},c_k)=W\left(\mathbf{y},c_k\right)d\mathbf{y}$ is the probability that the symbol c_k was transmitted and the observation vector was in volume $d\mathbf{y}$ relative to a point $\mathbf{y}$ in $2M$ -dimensional space simultaneously ($M=2^m$ complex observable numbers $\mathbf{y}$ are $2M=2^{m+1}$ real numbers and belong to the corresponding space), $W\left(\mathbf{y}\right)$ is a $2M$-dimensional unconditional probability density of the observation vector $\mathbf{y}$, $W\left(\mathbf{y}\right)d\mathbf{y}$ is a probability that the observation vector is in the volume $d\mathbf{y}$ relative to a point $\mathbf{y}$ in $2M$-dimensional space.

We can rewrite (8) as

$$P\left(c_k / \mathbf{y}\right)=\frac{W\left(\mathbf{y},c_k\right)d\mathbf{y}}{W\left(\mathbf{y}\right)d\mathbf{y}}=\frac{W\left(\mathbf{y} / c_k\right)P(c_k)}{\sum_{l=1}^{2^m}W\left(\mathbf{y} / c_l\right)P(c_l)}, \tag{10}$$

where $P(c_k)$, $k=1,\ldots,2^m$ are a priori probabilities of transmitting a symbol c_k from M available options, independent of the actions of the observer and the methods of processing the received signal, $W\left(\mathbf{y} / c_k\right)$ is a likelihood function of the hypothesis about the reception of a symbol c_k with the observed sample $\mathbf{y}$ (conditional probability density of the sample $\mathbf{y}$ when transmitting a symbol c_k).

Expression (10) is known as the inverse probability formula (Bayes' rule). The denominator of (10) uses the law of total probability.

$$W\left(\mathbf{y}\right)d\mathbf{y}=\sum_{l=1}^{2^m}W\left(\mathbf{y} / c_l\right)P(c_l)d\mathbf{y}\cdot \tag{11}$$

The values of the observation vector $\mathbf{y}=\left[\dot{y}_1,\dot{y}_2,\ldots,\dot{y}_{2^m}\right]^T$ are uncorrelated, due to the approximation of orthogonality of the signals (they are as coordinate functions in the orthogonal decomposition of a random process [25]), Gaussian complex variables (and therefore independent).

Therefore

$$W\left(\mathbf{y} / c_k\right)=\prod_{u=1}^{2^m}W\left(\dot{y}_u / c_k\right) \tag{12}$$

where $W\left(\dot{y}_u / c_k\right)$ is the likelihood function of the hypothesis about the transmission of a symbol c_k when observing $\dot{y}_u$ at the output of the processing device of the u-th variant of the orthogonal signal.

We assume that the transmission of any of the $M=2^m$ symbols is a priori equally probable, then the following is true:

$$P(c_k)=P(c_l)=\frac{1}{M}=\frac{1}{2^m}\quad \forall k,l \tag{13}$$

Taking into account (12) and (13), (10) can be rewritten as

$$P\left(c_k / \mathbf{y}\right)=\frac{W\left(\mathbf{y} / c_k\right)}{\sum_{l=1}^{2^m}W\left(\mathbf{y} / c_l\right)}=\frac{\prod_{u=1}^{2^m}W\left(\dot{y}_u / c_k\right)}{\sum_{l=1}^{2^m}\prod_{u=1}^{2^m}W\left(\dot{y}_u / c_l\right)}=$$
$$=\frac{\left(\prod_{\substack{u=1\\u\neq k}}^{2^m}W\left(\dot{y}_u / c_k\right)\right)W\left(\dot{y}_k / c_k\right)}{\sum_{l=1}^{2^m}\left[\left(\prod_{\substack{u=1\\u\neq l}}^{2^m}W\left(\dot{y}_u / c_l\right)\right)W\left(\dot{y}_l / c_l\right)\right]}, \tag{14}$$

where $W\left(\dot{y}_u / c_k\right)$ is the likelihood function of the hypothesis about the reception of a symbol c_k when observing $\dot{y}_u$ at the output of the u-th correlator. It is obvious that for non-coinciding indices u and k the likelihood function is determined by the noise probability density

$$W\left(\dot{y}_u / c_k\right)=W_n\left(\dot{y}_u\right)\ \text{при}\ u\neq k \tag{15}$$

Dividing the numerator by the denominator, and taking into account that 2^m-2 multipliers in each term of the denominator coincide with the multipliers of the numerator (with indices different from k and l), it can be obtained that

$$P\left(c_k / \mathbf{y}\right)=\frac{\dfrac{W\left(\dot{y}_k / c_k\right)}{W_n(\dot{y}_k)}}{\sum_{l=1}^{2^m}\left(\dfrac{W\left(\dot{y}_l / c_l\right)}{W_n(\dot{y}_l)}\right)} \tag{16}$$

Making hard decisions for each non-binary symbol using the maximum a posteriori probability criterion can be written as

$$\hat{c}_k=\underset{k=1,\ldots,2^m}{\arg\max}\left[P\left(c_k / \mathbf{y}\right)\right]=\underset{k=1,\ldots,2^m}{\arg\max}\left[\frac{W\left(\dot{y}_k / c_k\right)}{W_n\left(\dot{y}_k\right)}\right] \tag{17}$$

This corresponds to the criterion of maximum likelihood ratio, calculated for each variant of the transmitted symbol.

To simplify the calculations, formula (16) should be rewritten as

$$P\left(c_k / \mathbf{y}\right)=\frac{1.0}{\sum_{l=1}^{2^m}\left(\frac{W\left(\dot{y}_l / c_l\right)}{W_n(\dot{y}_l)}\frac{W_n(\dot{y}_k)}{W\left(\dot{y}_k / c_k\right)}\right)} \quad (18)$$

The denominator of the fraction (18) calculates the sum of the ratios of the likelihood ratios for each possible pair of symbols, one of which is the assumed received symbol c_k.

In multipath propagation conditions, the use of broadband signals allows the separation and addition of multipath components, which is a form of diversity reception. The diversity effect is achieved because individual multipath components propagate along different trajectories in the Earth's ionophore, reflecting off different layers of the ionosphere and undergoing birefringence due to the anisotropy of the environment.

Processing each *j*-th branch of the diversity (multipath component of the signal), the vector $\mathbf{y}_j$ is observed, and it is necessary to calculate the posterior probabilities for each symbol, taking into account observations from all branches of the diversity:

$$P\left(c_k / \mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_d\right)=\frac{W\left(\mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_d,c_k\right)}{W\left(\mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_d\right)}= \\ =\frac{W\left(\mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_d / c_k\right)P(c_k)}{\sum_{l=1}^{2^m}W\left(\mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_d / c_l\right)P(c_l)}. \quad (19)$$

Next it can be obtained

$$P\left(c_k / \mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_d\right)=\frac{\dfrac{W\left(\bar{\dot{y}}_k / c_k\right)}{W_n(\bar{\dot{y}}_k)}}{\sum_{l=1}^{2^m}\left(\dfrac{W\left(\bar{\dot{y}}_l / c_l\right)}{W_n(\bar{\dot{y}}_l)}\right)}= \\ =\frac{1.0}{\sum_{l=1}^{2^m}\left(\dfrac{W\left(\bar{\dot{y}}_l / c_l\right)}{W_n(\bar{\dot{y}}_l)}\dfrac{W_n(\bar{\dot{y}}_k)}{W\left(\bar{\dot{y}}_k / c_k\right)}\right)}, \quad (20)$$

where $\bar{\dot{y}}_l$ is the vector of responses of the *l*-th correlators for all branches of the diversity (values $\dot{y}_{l,1}, \dot{y}_{l,2}, ..., \dot{y}_{l,N_d}$), N_d is the number of branches of diversity (the number of multipath components), $W\left(\bar{\dot{y}}_l / c_l\right)$ is the likelihood function of the hypothesis about the reception of a symbol c_l during observations $\bar{\dot{y}}_l$, $W_n(\bar{\dot{y}}_l)$ is the joint probability density function of the noise responses of the *l*-th correlators of all N_d diversity branches.

In conditions of multi-channel diversity reception, taking into account the independence of noise in the channels, the following is true

$$\frac{W\left(\bar{\dot{y}}_l / c_l\right)}{W_n(\bar{\dot{y}}_l)}=\prod_{j=1}^{N_d}\frac{W\left(\dot{y}_{l,j} / c_l\right)}{W_n(\dot{y}_{l,j})}, \quad (21)$$

then

$$P\left(c_k / \mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_d\right)=\frac{\prod_{j=1}^{N_d}\dfrac{W\left(\dot{y}_{k,j} / c_k\right)}{W_n(\dot{y}_{k,j})}}{\sum_{l=1}^{2^m}\left(\prod_{j=1}^{N_d}\dfrac{W\left(\dot{y}_{l,j} / c_l\right)}{W_n(\dot{y}_{l,j})}\right)}= \\ =\frac{1.0}{\sum_{l=1}^{2^m}\left(\prod_{j=1}^{N_d}\left[\dfrac{W\left(\dot{y}_{l,j} / c_l\right)}{W_n(\dot{y}_{l,j})}\dfrac{W_n(\dot{y}_{k,j})}{W\left(\dot{y}_{k,j} / c_k\right)}\right]\right)}. \quad (22)$$

The decision rule (17) under conditions of multipath diversity reception can be rewritten as

$$\hat{c}_k=\underset{k=1,...,2^m}{\arg\max}\left[P\left(c_k / \mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_p\right)\right]=\underset{k=1,...,2^m}{\arg\max}\left[\frac{W\left(\bar{\dot{y}}_k / c_k\right)}{W_n(\bar{\dot{y}}_k)}\right]= \\ =\underset{k=1,...,2^m}{\arg\max}\left[\prod_{j=1}^{N_d}\frac{W\left(\dot{y}_{k,j} / c_k\right)}{W_n(\dot{y}_{k,j})}\right] \quad (23)$$

Particular expressions for calculating posterior probabilities

Coherent processing with fully known channel parameters

The likelihood function can be written as

$$W\left(\dot{y}_{k,j} / c_k\right)=W_n\left(\dot{y}_{k,j}-E_s\dot{h}_j\right)= \\ =\frac{1}{\sqrt{2\pi}\sigma_u}e^{-\frac{\left(y_{k,j,re}-E_sh_{j,re}\right)^2}{2\sigma_u^2}}\frac{1}{\sqrt{2\pi}\sigma_u}e^{-\frac{\left(y_{k,j,im}-E_sh_{j,im}\right)^2}{2\sigma_u^2}}= \\ =\left(\frac{1}{\sqrt{2\pi}\sigma_u}\right)^2e^{-\frac{\left|\dot{y}_{k,j}\right|^2-2E_s\,\mathrm{Re}\left(\dot{y}_{k,j}\dot{h}_j^*\right)+E_s^2\left|\dot{h}_j\right|^2}{2\sigma_u^2}} \quad (24)$$

The noise probability density calculated for the observations is

$$W_n\left(\dot{y}_{k,j}\right)=\frac{1}{\sqrt{2\pi}\sigma_u}e^{\frac{y_{k,j,re}^2}{2\sigma_u^2}}\frac{1}{\sqrt{2\pi}\sigma_u}e^{\frac{y_{k,j,im}^2}{2\sigma_u^2}}=\left(\frac{1}{\sqrt{2\pi}\sigma_u}\right)^2e^{\frac{\left|\dot{y}_{k,j}\right|^2}{2\sigma_u^2}}. \quad (25)$$

Then, the likelihood ratio can be written as

$$\frac{W\left(\dot{y}_{k,j} / c_k\right)}{W_n(\dot{y}_{k,j})}=e^{\frac{2E_s\,\mathrm{Re}\left(\dot{y}_{k,j}\dot{h}_j^*\right)-E_s^2\left|\dot{h}_j\right|^2}{2\sigma_u^2}}=e^{\frac{E_s\,\mathrm{Re}\left(\dot{y}_{k,j}\dot{h}_j^*\right)}{\sigma_u^2}}e^{-\frac{E_s^2\left|\dot{h}_j\right|^2}{2\sigma_u^2}}. \quad (26)$$

Taking into account the responses of the correlators for other multipath components we can obtain

$$\frac{W\left(\bar{\dot{y}}_k / c_k\right)}{W_n(\bar{\dot{y}}_k)}=\prod_{j=1}^{N_d}\frac{W\left(\dot{y}_{k,j} / c_k\right)}{W_n(\dot{y}_{k,j})}=e^{\sum_{j=1}^{N_d}\frac{E_s\,\mathrm{Re}\left(\dot{y}_{k,j}\dot{h}_j^*\right)}{\sigma_u^2}}e^{-\sum_{j=1}^{N_d}\frac{E_s^2\left|\dot{h}_j\right|^2}{2\sigma_u^2}} \quad (27)$$

The ratio of the likelihood ratios is equal to

$$\frac{W(\bar{\dot{y}}_l/c_l)}{W_n(\bar{\dot{y}}_l)}\frac{W_n(\bar{\dot{y}}_k)}{W(\bar{\dot{y}}_k/c_k)}=\prod_{j=1}^{N_d}\left[\frac{W(\dot{y}_{l,j}/c_l)}{W_n(\dot{y}_{l,j})}\frac{W_n(\dot{y}_{k,j})}{W(\dot{y}_{k,j}/c_k)}\right]=e^{\frac{\sum_{j=1}^{N_d}E_s\operatorname{Re}\left((\dot{y}_{l,j}-\dot{y}_{k,j})\dot{h}_j^*\right)}{\sigma_u^2}} \quad (28)$$

Taking hard decisions according to (23) and taking into account the monotonicity of the exponent leads to the rule

$$\hat{c}_k=\arg\max_{k=1,\dots,2^m}\left[P(c_k/\mathbf{y})\right]=\arg\max_{k=1,\dots,2^m}\left[\sum_{j=1}^{N_d}\operatorname{Re}\left(\dot{y}_{k,j}\dot{h}_j^*\right)\right]. \quad (29)$$

Decision rule (29) is known as the optimal coherent diversity reception rule, which ensures the maximum signal-to-noise ratio (in English literature, the MRC algorithm – maximum ratio combining). To use rule (29), the information about the noise variance and the channel multiplier values for each multipath component $\dot{h}_j$ must be available. The channel multiplier $\dot{h}_j$ determines the change in signal amplitude by $|\dot{h}_j|$ and the signal phase shift by $\varphi_{h,j}=\arg\left(\dot{h}_j\right)$ due to fading.

Incoherent processing with unknown phase shift

In cases when it is not possible to measure the phase shift and use its measured value in the expressions written above, decision rules and formulas for calculating posterior probabilities that are invariant to the value of the phase shift are used. The method for synthesizing such rules is well known and widely used in statistical radio engineering [26].

The likelihood ratio (26) is conditional, where the phase shift $\varphi_{h,j}$ is one of conditions, so we can write

$$\frac{W(\dot{y}_{k,j}/c_k,\varphi_{h,j})}{W_n(\dot{y}_{k,j})}=e^{\frac{E_s\operatorname{Re}(\dot{y}_{k,j}\dot{h}_j^*)}{\sigma_u^2}}e^{-\frac{E_s^2|\dot{h}_j|^2}{2\sigma_u^2}}=e^{\frac{E_s|\dot{h}_j||\dot{y}_{k,j}|\cos(\arg(\dot{y}_{k,j})-\varphi_{h,j})}{\sigma_u^2}}e^{-\frac{E_s^2|\dot{h}_j|^2}{2\sigma_u^2}}. \quad (30)$$

The phase shift $\varphi_{h,j}$ is assumed to be a random variable with some probability density $W_\varphi(\varphi_{h,j})$. Then the joint conditional probability density of the observation $\dot{y}_k$ and the value of the phase shift $\varphi_{h,j}$ during symbol transmission c_k can be written in the form

$$W\left(\dot{y}_{k,j},\varphi_{k,j}/c_k\right)=W\left(\dot{y}_{k,j}/c_k,\varphi_{k,j}\right)W_\varphi\left(\varphi_{k,j}\right). \quad (31)$$

Therefore, using the consistency rule, we can get rid of the dependence on the phase shift $\varphi_{h,j}$ by integrating over it:

$$W\left(\dot{y}_{k,j}/c_k\right)=\int_{-\pi}^{\pi}W\left(\dot{y}_k,\varphi_{h,j}/c_k\right)d\varphi_{h,j}= \quad (32)$$

$$=\int_{-\pi}^{\pi}W\left(\dot{y}_{k,j}/c_k,\varphi_{h,j}\right)W_\varphi\left(\varphi_{h,j}\right)d\varphi_{h,j}=\left\langle W\left(\dot{y}_{k,j}/c_k,\varphi_{h,j}\right)\right\rangle,$$

where the symbol $\langle\ \rangle$ means averaging. Indeed, the conditional probability density (24), as a function of the phase shift $\varphi_{h,j}$, is averaged over $\varphi_{h,j}$ in the sense of finding the mathematical expectation taking into account the probability density $W_\varphi\left(\varphi_{h,j}\right)$.

Likewise, the likelihood ratio can be averaged (30)

$$\frac{W\left(\dot{y}_{k,j}/c_k\right)}{W_n(\dot{y}_{k,j})}=\left\langle\frac{W\left(\dot{y}_{k,j}/c_k,\varphi_{h,j}\right)}{W_n(\dot{y}_{k,j})}\right\rangle=. \quad (33)$$

$$=\int_{-\pi}^{\pi}\frac{W\left(\dot{y}_{k,j}/c_k,\varphi_{h,j}\right)}{W_n(\dot{y}_{k,j})}W_\varphi\left(\varphi_{h,j}\right)d\varphi_{h,j}$$

The average likelihood ratio can be found according to (33) taking into account that the initial phase is distributed uniformly. It means that

$$W_\varphi\left(\varphi_{h,j}\right)=\frac{1}{2\pi},\ -\pi\le\varphi_0<\pi. \quad (34)$$

Then, it can be obtained that

$$\frac{W\left(\dot{y}_{k,j}/c_k\right)}{W_n(\dot{y}_{k,j})}=\int_{-\pi}^{\pi}\frac{W\left(\dot{y}_{k,j}/c_k,\varphi_{h,j}\right)}{W_n(\dot{y}_{k,j})}W_\varphi\left(\varphi_{h,j}\right)d\varphi_{h,j}$$

$$=e^{-\frac{E_s^2|\dot{h}_j|^2}{2\sigma_u^2}}\frac{1}{2\pi}\int_0^{2\pi}e^{\frac{E_s|\dot{h}_j||\dot{y}_{k,j}|\cos(\arg(\dot{y}_{k,j})-\varphi_{h,j})}{\sigma_u^2}}d\varphi_{h,j}= \quad . \quad (35)$$

$$=e^{-\frac{E_s^2|\dot{h}_j|^2}{2\sigma_u^2}}I_0\left(\frac{E_s|\dot{h}_j||\dot{y}_{k,j}|}{\sigma_u^2}\right).$$

In (35) the integral representation of the modified Bessel function of the first kind of zero order was used in the form

$$I_0(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{x\cos\varphi_0}d\varphi_0. \quad (36)$$

The decision rule can be obtained relying on (23) as

$$\hat{c}_k=\arg\max_{k=1,\dots,2^m}\left[P(c_k/\mathbf{y}_1,\mathbf{y}_2,\dots,\mathbf{y}_d)\right]=$$

$$=\arg\max_{k=1,\dots,2^m}\left[\prod_{j=1}^{N_d}e^{-\frac{E_s^2|\dot{h}_j|^2}{2\sigma_u^2}}I_0\left(\frac{E_s|\dot{h}_j||\dot{y}_{k,j}|}{\sigma_u^2}\right)\right]. \quad (37)$$

The ratio of the likelihood ratios can be written as

$$\frac{W(\bar{\dot{y}}_l/c_l)}{W_n(\bar{\dot{y}}_l)}\frac{W_n(\bar{\dot{y}}_k)}{W(\bar{\dot{y}}_k/c_k)}=\prod_{j=1}^{N_d}\left(I_0\left(\frac{E_s|\dot{h}_j||\dot{y}_{l,j}|}{\sigma_u^2}\right)\left[I_0\left(\frac{E_s|\dot{h}_j||\dot{y}_{k,j}|}{\sigma_u^2}\right)\right]^{-1}\right) \quad (38)$$

Incoherent processing with unknown phase shift and averaging over signal level

Consider modules of channel coefficients $|\dot{h}_j|$ as random variables with a Rayleigh distribution with a distribution parameter σ_{Aj}^2:

$$W_{hj}\left(|\dot{h}_j|\right)=\frac{|\dot{h}_j|}{\sigma_{Aj}^2}e^{-\frac{|\dot{h}_j|^2}{2\sigma_{Aj}^2}}. \quad (39)$$

Next, the likelihood ratio should be averaged (35) over $|\dot{h}_j|$. So, we can obtain

$$\frac{W\left(\dot{y}_{k,j}/c_k\right)}{W_n(\dot{y}_{k,j})}=\int_0^{\infty}\frac{W\left(\dot{y}_{k,j}/c_k,\left|\dot{h}_j\right|\right)}{W_n(\dot{y}_{k,j})}W_{hj}\left(\left|\dot{h}_j\right|\right)d\left|\dot{h}_j\right|= \quad (40)$$
$$=\frac{1}{2\sigma_{Aj}^2}e^{\left|\dot{y}_{k,j}\right|^2\frac{\sigma_{Aj}^2E_s^2}{2\sigma_u^2\left(\sigma_{Aj}^2E_s^2+\sigma_u^2\right)}}.$$

Taking into account (40), the expression (21) can be rewritten

$$\frac{W\left(\bar{y}_l/c_l\right)}{W_n(\bar{y}_l)}=\prod_{j=1}^{N_d}\frac{W\left(\dot{y}_{l,j}/c_l\right)}{W_n(\dot{y}_{l,j})}=\prod_{j=1}^{N_d}\frac{1}{2\sigma_{Aj}^2}e^{\left|\dot{y}_{k,j}\right|^2\frac{\sigma_{Aj}^2E_s^2}{2\sigma_u^2\left(\sigma_{Aj}^2E_s^2+\sigma_u^2\right)}}. \quad (41)$$

Thus

$$\frac{W\left(\dot{y}_{l,j}/c_l\right)}{W_n(\dot{y}_{l,j})}\frac{W_n(\dot{y}_{k,j})}{W\left(\dot{y}_{k,j}/c_k\right)}=e^{-\left(\left|\dot{y}_{k,j}\right|^2-\left|\dot{y}_{l,j}\right|^2\right)\frac{\sigma_{Aj}^2E_s^2}{2\sigma_u^2\left(\sigma_{Aj}^2E_s^2+\sigma_u^2\right)}}, \quad (42)$$

and

$$\frac{W\left(\bar{y}_l/c_l\right)}{W_n(\bar{y}_l)}\frac{W_n(\bar{y}_k)}{W\left(\bar{y}_k/c_k\right)}=e^{-\sum_{j=1}^{N_d}\left(\left|\dot{y}_{k,j}\right|^2-\left|\dot{y}_{l,j}\right|^2\right)\frac{\sigma_{Aj}^2E_s^2}{2\sigma_u^2\left(\sigma_{Aj}^2E_s^2+\sigma_u^2\right)}}. \quad (43)$$

Hence, according to (23), the decision rule can be written as

$$\hat{c}_k=\underset{k=1,\dots,2^m}{\arg\max}\left[P\left(c_k/\mathbf{y}_1,\mathbf{y}_2,\dots,\mathbf{y}_d\right)\right]=$$
$$=\underset{k=1,\dots,2^m}{\arg\max}\left[\prod_{j=1}^{N_d}\frac{1}{2\sigma_{Aj}^2}e^{\left|\dot{y}_{k,j}\right|^2\frac{\sigma_{Aj}^2E_s^2}{2\sigma_u^2\left(\sigma_{Aj}^2E_s^2+\sigma_u^2\right)}}\right]. \quad (44)$$

If the fading in all beams has the same intensity, then $\sigma_{Aj}^2=\sigma_A^2,\ \forall j=1,\dots,N_d$ and the decision rule is to find the maximum of the sum of the squared modules of the correlator responses

$$\hat{c}_k=\underset{k=1,\dots,2^m}{\arg\max}\left[e^{\frac{\sigma_A^2E_s^2}{2\sigma_u^2\left(\sigma_A^2E_s^2+\sigma_u^2\right)}\sum_{j=1}^{N_d}\left|\dot{y}_{k,j}\right|^2}\right]=\underset{k=1,\dots,2^m}{\arg\max}\left[\sum_{j=1}^{N_d}\left|\dot{y}_{k,j}\right|^2\right]. \quad (45)$$

Incoherent processing with quadratic addition

Quadratic addition according to (45) is used when the distribution law of $\left|\dot{h}_j\right|$ is a priori unknown. In this case, it is the optimal decision-making option [24].

Using the observation model, centered and non-centered chi-square distributions, expressions for the likelihood ratio can be obtained as [6-8]

$$\frac{W\left(\bar{y}_l/c_l\right)}{W_n(\bar{y}_l)}\frac{W_n(\bar{y}_k)}{W\left(\bar{y}_k/c_k\right)}=\left(\frac{\left|\dot{y}_l\right|^2}{\left|\dot{y}_k\right|^2}\right)^{\frac{1-N_d}{2}}I_{N_d-1}\left(\sqrt{\frac{\left|\dot{y}_l\right|^2}{\sigma_u^2}\frac{E_s^2\sum_{j=1}^{N_d}\left|\dot{h}_j\right|^2}{\sigma_u^2}}\right)\times \quad (46)$$
$$\times\left[I_{N_d-1}\left(\sqrt{\frac{\left|\dot{y}_k\right|^2}{\sigma_u^2}\frac{E_s^2\sum_{j=1}^{N_d}\left|\dot{h}_j\right|^2}{\sigma_u^2}}\right)\right]^{-1}$$

where $I_{N_d-1}(x)$ is the modified Bessel function of the first kind and N_d-1 order,

$$\left|\dot{y}_k\right|^2=\sum_{j=1}^{N_d}\left|\dot{y}_{k,j}\right|^2,\quad \left|\dot{y}_l\right|^2=\sum_{j=1}^{N_d}\left|\dot{y}_{l,j}\right|^2. \quad (47)$$

Clearly that for $N_d=1$ the expression (46) transfers to (38).

The algorithm for coherent processing of non-binary wideband signal-code structures with simultaneous estimation of channel coefficients

Observation model for radiogram reception

The length of the radiogram is N non-binary symbols, of which N_{pr} symbols are the known symbols of preamble and N_{data} information (data) symbols, and

$$N=N_{pr}+N_{data}. \quad (48)$$

Preamble symbols are required for radiogram detection and synchronization. So, they are transmitted before the data symbols. The parameters of the radiogram and signal-code structures are presented below [5]. The data transfer rate corresponds to the vocoder rate – 700 bit/s. The radiogram length is 160 ms, the symbol capacity m = 6, the number of preamble symbols N_{pr} = 16, the number of data symbols N_{data} = 44, and the total number of radiogram symbols N = 60. The relative coding rate is $r=0.5$. The length of the pseudo-random sequences (PRS), the PRS symbol transfer rate, and the spectrum width of the signal-code structure are presented in Table 1.

Table 1

Calculated data of wideband signals based on the NB-LDPC in the 80 kHz band

№	Symbol transfer rate, kBod	Spectral width, kHz	Length of the pseudo-random sequences
1	50	80	128
2	100	160	256
3	200	320	512

The model of the received signal in the single-path case can be written as

$$\dot{y}(k,n)=\delta(k,d(n))\dot{h}(n)E_s+\dot{u}(k,n),\ n=0,\dots,N-1,$$
$$k=0,\dots,2^m-1 \quad (49)$$

where $\dot{y}(k,n)$ is the matrix of complex responses of correlators, in general case the size of the matrix is $M\times N$, $\dot{u}(k,n)$ is the matrix of complex Gaussian noise samples at the output of the correlators, the size is $M\times N$, $\dot{h}(n)$ is a complex channel gain, $d(n)$ is a transmitted non-binary symbol (from 0 to 2^m-1) at time n,

$$\delta(k,n)=\begin{cases}1,k=n\\0,k\neq n\end{cases} \quad (50)$$

– ronecker delta. The noise samples, as before, are centered, independent in different correlator responses, at different times and in two channels (real and imaginary components), with the same dispersion σ_u^2.

Thus, a matrix of numbers $\dot{y}(k,n)$ is processed at the reception point. The matrix $\dot{y}(k,n)$ contains noise samples, and in each column of the matrix, one value contains the signal component of the response $\dot{h}(n)E_s$. The position of the response in the corresponding column (i.e., the row number) is determined by the transmitted symbol $d(n)$ at the corresponding time *n* (which coincides with the column number).

If there are N_d multipath components, the observation model will include N_d matrices of size $M \times N$:

$$\dot{y}_j(k,n) = \delta(k,d(n))\dot{h}_j(n)E_s + \dot{u}_j(k,n),$$
$$n = 0,...,N-1,\ k = 0,...,2^m - 1,\ j = 1,...,N_d \quad (51)$$

The main goal of the article is to synthesize an algorithm for processing the received sample with simultaneous evaluation of channel coefficients $\dot{h}(n)$, making "hard" decisions regarding the transmitted symbols $d(n)$ and calculating the posterior probabilities discussed above.

The process $\dot{h}_j(n)$ is assumed to vary sufficiently slowly according to the Waterson channel model that a Kalman filter can be applied to refine maximum likelihood estimates of $\dot{h}(n)$ during preamble processing and subsequent extrapolation at the time of reception of data symbols.

In the absence of any information about the channel coefficients $\dot{h}(n)$, non-coherent demodulation can be performed by finding the maximum among the sum of the squares of the correlator responses at a fixed point in time according to (45), i.e.

$$\hat{d}(n) = \underset{k=0,...,2^m-1}{\arg\max}\left(\sum_{j=1}^{N_d}\left|\dot{y}_j(k,n)\right|^2\right). \quad (52)$$

Algorithm (52) is optimal for Rayleigh fading and for an unknown fading distribution if $\sigma_{A,j}^2$ are the same.

If channel coefficients $\dot{h}_j(n)$ are reliably known, the decision rule can be formulated in the form of a coherent demodulation algorithm according to (29)

$$\hat{d}(n) = \underset{k=0,...,2^m-1}{\arg\max}\left(\mathrm{Re}\left(\sum_{j=1}^{N_d}\dot{y}_j(k,n)\dot{h}_j^*(n)\right)\right). \quad (53)$$

Thus, the values of $\dot{h}_j(n)$ have to be known in order to perform coherent demodulation that provides better noise immunity. Therefore, it is necessary to estimate channel coefficients $\dot{h}_j(n)$ during the demodulation process.

Algorithm for estimating channel coefficients using the maximum likelihood criterion

Additionally, the presence of a separate channel for measuring the coefficients $\dot{h}_j(n)$ for each multipath component is assumed on order to compare different processing options. At the output of the channel, we can observe

$$\dot{y}_{j,measure}(n) = \dot{h}_j(n)E_s + \dot{u}_{j,measure}(n),\ n = 0,...,N-1, \quad (54)$$

where $\dot{u}_{j,measure}(n)$ is complex samples of Gaussian noise. Noise samples $\dot{u}_{j,measure}(n)$ are centered, the variance of the real and imaginary parts is the same as that of the samples $\dot{u}(k,n)$.

The maximum likelihood estimation of the channel coefficients using the measurement channel (54) is determined as

$$\hat{\dot{h}}_j(n) = \frac{\dot{y}_{j,measure}(n)}{E_s},\ n = 0,...,N-1 \quad (55)$$

The algorithm for estimating channel coefficients using a Kalman filter based on measurement channel data

The simplest dynamic model of change of the coefficients $\dot{h}(n)$ can be written as

$$\dot{h}_j(n) = \rho\dot{h}_j(n-1) + \dot{\xi}_j(n), \quad (56)$$

where ρ is a correlation coefficient of channel coefficients $\dot{h}_j(n)$ at adjacent points in time, $\dot{\xi}_j(n)$ is noise of a dynamic system with variance $\sigma_{\xi,j}^2$ (Gaussian centered, uncorrelated), and

$$\sigma_{\xi,j}^2 = \left(1-\rho^2\right)\sigma_{A,j}^2, \quad (57)$$

where $\sigma_{A,j}^2$ is a variance of real and imaginary parts of $\dot{h}_j(n)$ for each $n = 0,...,N-1$. The initial value of the channel coefficient $\dot{h}_j(0)$ is calculated as

$$\dot{h}(0) = \frac{\sigma_{A,j}}{\sigma_{\xi,j}}\dot{\xi}(0). \quad (58)$$

Using the measurement channel of channel gains (54) and the dynamic model (56), we can perform optimal filtering of the channel coefficients $\dot{h}_j(n)$ for each multipath component independently and then perform coherent demodulation according to (53) utilizing the obtained estimates.

The vector-matrix notations which are conventional to optimal filtering theory are introduced below

$$\mathbf{H} = E_s,\ \mathbf{F} = \rho, \quad (59)$$

– 1x1 matrices,

$$\mathbf{h}_{n,j} = \dot{h}_j(n),\ \mathbf{y}_{n,j} = \dot{y}_{j,measure}(n), \quad (60)$$

– complex vectors of size 1x1, depending on the moment of time $n = 0,...,N-1,\ j = 1,...,N_d$

$$\mathbf{Q}_j = \sigma_{\xi,j}^2,\ \mathbf{R} = \sigma_u^2, \quad (61)$$

– 1x1 matrices describing the variance of noise of the dynamic system and observations, respectively, $j = 1,...,N_d$.

Then filtering the channel coefficients $\dot{h}_j(n)$ for each j-th multipath component is consist of performing the following sequence of actions.

Predicting the value of the channel coefficient estimation based on the dynamic model and the estimation obtained in the previous step

$$\hat{\mathbf{h}}_{n,j}^{-} = \mathbf{F}\hat{\mathbf{h}}_{n-1,j}. \quad (62)$$

Prediction of the covariance (error) matrix of the channel coefficient estimation, based on the estimated value of the covariance (error) matrix of the channel coefficient estimation obtained in the previous step

$$\hat{\mathbf{P}}_{n,j}^{-} = \mathbf{F}\hat{\mathbf{P}}_{n-1,j}\mathbf{F}^{T} + \mathbf{Q}_{j}. \quad (63)$$

Calculating the Kalman filter gain matrix

$$\mathbf{K}_{n,j} = \hat{\mathbf{P}}_{n,j}^{-}\mathbf{H}^{T}\left(\mathbf{H}\ \hat{\mathbf{P}}_{n,j}^{-}\mathbf{H}^{T} + \mathbf{R}\ \right)^{-1}. \quad (64)$$

Prediction of the expected observations at the current step

$$\hat{\mathbf{y}}_{n,j}^{-} = \mathbf{H}\hat{\mathbf{h}}_{n,j}^{-}. \quad (65)$$

Refinement of the estimation by adding a weighted residual of observations to the previously predicted estimation value

$$\hat{\mathbf{h}}_{n,j} = \hat{\mathbf{h}}_{n,j}^{-} + \mathbf{K}_{n,j}\left(\mathbf{y}_{n,j} - \hat{\mathbf{y}}_{n,j}^{-}\right). \quad (66)$$

Updating the covariance (error) matrix of the channel coefficient estimation

$$\hat{\mathbf{P}}_{n,j} = \left(\mathbf{I} - \mathbf{K}_{n,j}\mathbf{H}\right)\hat{\mathbf{P}}_{n,j}^{-}. \quad (67)$$

Initial conditions are

$$\hat{\mathbf{h}}_{0,j} = \hat{\dot{h}}_j(0) = \frac{\dot{y}_{j,measure}(0)}{E_s} \quad (68)$$

– ML estimation of the channel coefficient $\dot{h}_j(0)$ at the initial moment of time,

$$\hat{\mathbf{P}}_{0,j} = \frac{\sigma_u^2}{E_s^2} \quad (69)$$

– the variance of ML estimation of the channel coefficient $\dot{h}_j(0)$ at the initial moment of time (real and imaginary components).

The algorithm for estimating channel coefficients using a Kalman filter based on data symbols

Filtering channel coefficients directly from the radiogram (radiogram includes preambles and data symbols) has its own points and features since data symbols are unknown in contrast with symbols of the preamble. As preamble symbols are known, using them allows selecting the correlator response, which contains the signal component at the current moment, from the observation matrix (49).

Filtering channel coefficients directly from the radiogram can be described using following sequence of steps.

Predicting the channel coefficient estimation based on the dynamic model and the estimation obtained in the previous step

$$\hat{\mathbf{h}}_{n,j}^{-} = \mathbf{F}\hat{\mathbf{h}}_{n-1,j}. \quad (70)$$

Prediction of the covariance (error) matrix of the channel coefficient estimation, based on the estimated value of the covariance (error) matrix of the channel coefficient estimation obtained in the previous step

$$\hat{\mathbf{P}}_{n,j}^{-} = \mathbf{F}\hat{\mathbf{P}}_{n-1,j}\mathbf{F}^{T} + \mathbf{Q}_{j}. \quad (71)$$

Calculating the Kalman filter gain matrix

$$\mathbf{K}_{n,j} = \hat{\mathbf{P}}_{n,j}^{-}\mathbf{H}^{T}\left(\mathbf{H}\ \hat{\mathbf{P}}_{n,j}^{-}\mathbf{H}^{T} + \mathbf{R}\ \right)^{-1}. \quad (72)$$

Prediction of the expected observations at the current step

$$\hat{\mathbf{y}}_{n,j}^{-} = \mathbf{H}\hat{\mathbf{h}}_{n,j}^{-}. \quad (73)$$

If a symbol of preamble is processed, the observation values from a known row of the observation matrix are selected

$$\mathbf{y}_{n,j} = \dot{y}_j(d(n),n). \quad (74)$$

If a data symbol is processed, a decision is made using the predictive estimation of the channel coefficient according to (70) as

$$\hat{d}^{-}(n) = \underset{k=0,\dots,2^m-1}{\arg\max}\left(\mathrm{Re}\left(\sum_{j=1}^{N_d}\dot{y}_j(k,n)\left(\hat{\dot{h}}_j^{-}(n)\right)^{*}\right)\right),$$
$$\hat{\dot{h}}_j^{-}(n) \equiv \hat{\mathbf{h}}_{n,j}^{-} \quad (75)$$

Then the appropriate measurement is selected in order to refine the estimation

$$\mathbf{y}_{n,j} = \dot{y}_j(\hat{d}^{-}(n),n). \quad (76)$$

Refinement of the estimation by adding a weighted residual of observations to the previously predicted estimation value

$$\hat{\mathbf{h}}_{n,j} = \hat{\mathbf{h}}_{n,j}^{-} + \mathbf{K}_{n,j}\left(\mathbf{y}_{n,j} - \hat{\mathbf{y}}_{n,j}^{-}\right). \quad (77)$$

Updating the covariance (error) matrix of the channel coefficient estimation

$$\hat{\mathbf{P}}_{n,j} = \left(\mathbf{I} - \mathbf{K}_{n,j}\mathbf{H}\right)\hat{\mathbf{P}}_{n,j}^{-}. \quad (78)$$

The estimations of channel coefficients obtained according to (66) or (77) are used for coherent demodulation according to (53).

Comparison of noise immunity of signal processing algorithms

Using simulation, four different options for processing the observed sample without taking into account error-correcting coding have been compared:

– a coherent demodulation algorithm with known channel gains according to (53);

– a non-coherent demodulation algorithm with unknown channel gains according to (52);

– a coherent demodulation algorithm (52) taking into account the channel coefficient estimation according to (55) using the maximum likelihood method;

– a coherent demodulation algorithm with channel coefficient estimation $\dot{h}(n)$ using a Kalman filter based on preamble symbols and data symbols.

Figure 1 shows the symbol error rate (SER) in a single-path channel in case of estimating channel coefficients directly from radiogram symbols – from preamble symbols and from data symbols (taking into account decision-making on them).

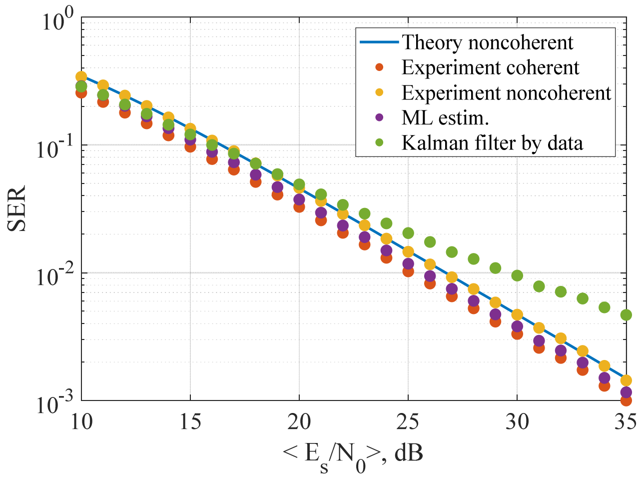


Fig. 1. SER, the estimation of channel coefficients obtained from preamble data symbols

The figure shows that the estimation of channel coefficients based on a radiogram significantly loses noise immunity of information reception due to errors in decision-making, which affect the quality of the estimation (which in turn affects the quality of decision-making on data symbols).

The Watterson channel coefficients are not ordinary Markov sequences, so their statistical properties do not correspond to first-order autoregression. Indeed, the correlation function of a Gaussian Markov process is described by an exponentially decaying function, whereas the Watterson channel model assumes a Gaussian function as the correlation function.

Thus, the model of the dynamic system used in the filtering algorithm does not match the channel model, which is a source of filtering errors [27]. Furthermore, first-order autoregression allows predicting the channel coefficient using only a single previous channel coefficient estimation. If the previous estimation deviates significantly from the true value due to an error in the data symbol, the quality of the current channel coefficient prediction based solely on this previous estimation will also be poor.

It can be concluded that the construction of autoregression model of a higher order is necessary in order to satisfy the Watterson channel model.

Formation of the autoregressive observation model and justification of its parameters

One of the options for making the dynamic model closer to the properties of a real process is to increase the order of autoregression to $N_{ar} > 1$, so that

$$\dot{h}_j(n) = \sum_{k=1}^{N_{ar}} \alpha_k \dot{h}_j(n-k) + \dot{\xi}_j(n), \tag{79}$$

where α_k are autoregression coefficients, N_{ap} is the order of autoregression.

To increase the convenience of mathematical description, we should switch to vector-matrix notation. We define the vectors of the current and previous states in the form of vectors of size $N_{ar} \times 1$:

$$\begin{aligned} \mathbf{h}_{n,j} &= \left[\dot{h}_j(n), \dot{h}_j(n-1), ..., \dot{h}\,(n-N_{ar}+1) \right]^T, \\ \mathbf{h}_{n-1,j} &= \left[\dot{h}_j(n-1), \dot{h}_j(n-2), ..., \dot{h}_j(n-N_{ar}) \right]^T. \end{aligned} \tag{80}$$

Hence, the dynamic model can be written as

$$\mathbf{h}_{n,j} = \mathbf{F}\mathbf{h}_{n-1,j} + \mathbf{e}_{n,j}, \tag{81}$$

where

$$\mathbf{F} = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & ... & \alpha_{N_{ar}-1} & \alpha_{N_{ar}} \\ 1 & 0 & 0 & ... & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 \\ 0 & 0 & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 & 0 \end{pmatrix}, \tag{82}$$

is the matrix which size is $N_{ar} \times N_{ar}$,

$$\mathbf{e}_{n,j} = \left[\dot{\xi}_j(n), 0, 0, ..., 0 \right]^T, \tag{83}$$

is the vector which size is $N_{ar} \times 1$.

The coefficients α_k can be found through the solution of the Yule-Walker equation:

$$\mathbf{R}\boldsymbol{\alpha} = \boldsymbol{\rho}, \tag{84}$$

where

$$\boldsymbol{\rho} = \left[\rho_1, \rho_2, \rho_3, ..., \rho_{N_{ar}} \right]^T, \tag{85}$$

is the vector of correlation coefficients ρ_k between real (or imaginary) parts of $\dot{h}_j(n)$ and $\dot{h}_j(n-k)$,

$$\boldsymbol{\alpha} = \left[\alpha_1, \alpha_2, \alpha_3, ..., \alpha_{N_{ar}} \right]^T, \tag{86}$$

is the vector of the required coefficients,

$$\mathbf{R} = \begin{pmatrix} 1 & \rho_1 & \rho_2 & ... & \rho_{N_{ar}-2} & \rho_{N_{ar}-1} \\ \rho_1 & 1 & \rho_1 & ... & \rho_{N_{ar}-3} & \rho_{N_{ar}-2} \\ \rho_2 & \rho_1 & 1 & ... & \rho_{N_{ar}-4} & \rho_{N_{ar}-3} \\ \rho_3 & \rho_2 & \rho_1 & ... & \rho_{N_{ar}-5} & \rho_{N_{ar}-4} \\ ... & ... & ... & ... & ... & ... \\ \rho_{N_{ar}-1} & \rho_{N_{ar}-2} & \rho_{N_{ar}-3} & ... & \rho_1 & 1 \end{pmatrix}, \tag{87}$$

is equation matrix.

The solution of the equation (84) can be obtained as

$$\boldsymbol{\alpha} = \mathbf{R}^{-1}\boldsymbol{\rho}. \tag{88}$$

Next, we will find the coefficients for the fourth-order autoregression for the Watterson channel. For simplicity, we assume that the magneto-ionic components are separated ideally, and the frequency shift can be neglected. So, the expression for the correlation function can be simplified to

$$C(\Delta t) = e^{-2\pi^2\sigma_{sia}^2(\Delta t)^2}, \tag{89}$$

and the spectral power density of the random process that models fading [23], respectively, to

$$S_c(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{sia}} e^{-\frac{\upsilon^2}{2\sigma_{sia}^2}} \tag{90}$$

Thus, taking into account (89), it can be obtained that

$$\rho_k = C(kT_s) = e^{-2\pi^2\sigma_{sia}^2(kT_s)^2}. \tag{91}$$

Calculations can show that the autoregression that directly exploits coefficients calculated according to (87) – (91) is unstable, i.e., to an unstable IIR filter whose coefficients are the autoregression coefficients. This can be explained by the fact that the filter that provides an exact match between the square of its amplitude-frequency response (AFR) and (90) is not physically feasible. The AFR of such a filter is described by a Gaussian function, and, consequently, the impulse response is be described by a Gaussian function that is not bounded on the abscissa.

Before forming the autoregressive model and calculating its coefficients, a model for calculating the channel coefficients in the form of a moving average should be developed. The coefficients in the model are a finite number of samples of the filter impulse response, providing a power spectral density of the generated random process close to (90).

Therefore, we can obtain that

$$\rho_k = \frac{\sum_{n=0}^{L-k} b_n b_{n+k}}{\sum_{j=0}^{L} b_n^2}, \tag{92}$$

where L is a number of mentioned coefficients (odd number), b_j, $j = 0,...,L$ – filter impulse response coefficients determined by the formula

$$b_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} \sqrt{S_c(\nu)} e^{j\nu(n-(L-1)/2)T_s} d\nu. \tag{93}$$

The coefficients b_j can also be obtained using frequency sampling algorithm by sampling the filter's frequency response $\sqrt{S_c(\nu)}$ in the frequency domain and calculating the inverse fast Fourier transform.

Thus, using (92), instead of (91), allows obtaining stable autoregressive models.

The autoregressive model allows calculating the variance of the dynamic system noise for the required variance of the observed channel coefficients at the autoregressive output (which determines the signal level and signal-to-noise ratio). Indeed, the autoregressive model can be represented as a recursive filter at the input of which is the noise of the dynamic system (see Fig. 2).

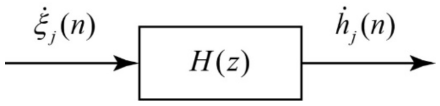


Fig. 2. Autoregressive model as a recursive filter

The transfer function of the filter and its complex frequency response are determined by the autoregressive coefficients, respectively, as

$$H(z) = \frac{1}{1 - \sum_{k=1}^{N_{ar}} \alpha_k z^{-k}}, \tag{94}$$

$$\dot{H}(e^{j\hat{\omega}}) = \frac{1}{1 - \sum_{k=1}^{N_{ar}} \alpha_k e^{-j\hat{\omega}k}}, \tag{95}$$

where $\hat{\omega}$ is normalized frequency. Assuming the noise of the dynamic system to be white in the main frequency range and Gaussian, the variance of the noise can be written as (its real and imaginary parts)

$$\sigma_{\xi,j}^2 = \frac{N_{\xi 0,j}}{2} F_s, \tag{96}$$

where $\frac{N_{\xi 0,j}}{2}$ is two-sided spectral power density of the noise of the dynamic system $F_s = 1/T_s$ is the sampling rate, with which samples of channel coefficients are generated (equal to the bit rate).

The dispersion of the formed channel coefficients (real and imaginary parts) is determined by the expression

$$\begin{aligned} \sigma_{A,j}^2 &= \frac{N_{\xi 0,j}}{2}\frac{1}{2\pi}\int_{-\omega_s/2}^{\omega_s/2} \left|\dot{H}(e^{j\omega T_s})\right|^2 d\omega = \\ &= \frac{N_{\xi 0,j}}{2} F_s \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|\dot{H}(e^{j\hat{\omega}})\right|^2 d\hat{\omega} = \\ &= \sigma_{\xi,j}^2 \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|\dot{H}(e^{j\hat{\omega}})\right|^2 d\hat{\omega} \end{aligned} \tag{97}$$

where $\omega_s = 2\pi F_s$.

Then the variance of the noise of the dynamic system for a given variance of the channel coefficients can be calculated using the formula

$$\sigma_{\xi,j}^2 = \sigma_{A,j}^2 \frac{2\pi}{\frac{1}{2\pi}\int_{-\pi}^{\pi} \left|\dot{H}(e^{j\hat{\omega}})\right|^2 d\hat{\omega}} \tag{98}$$

It should be noted that autoregression of order $q > 1$ has two advantages:

– predicting the next value of the filtered parameter $\hat{\dot{h}}_j^-(n)$ is carried out using $q>1$ previous estimations $\hat{\dot{h}}_j(n-1)$, $\hat{\dot{h}}_j(n-2)$ $,\ldots,\hat{\dot{h}}_j(n-q)$;

– the estimation of $\hat{\dot{h}}_j(n)$ taking into account the refinement according to (66) (or (77)) is carried out sequentially according to $q>1$ the measurements $\dot{y}_j(k,n)$, $\dot{y}_j(k,n+1),\ldots,$ $\dot{y}_j(k,n+q-1)$, where k is the number of the index of the decision made (or the preamble number).

The main disadvantage of this approach is the significant increase in computational complexity, due to the dimensions of the vectors and matrices included in the main expressions of the Kalman filter algorithm. However, it should be noted, that the computational complexity does not increase as much as might be expected. For example, in (64) and (72), the increase in the autoregressive order does not lead to the increase in the dimension of the inverted matrix. In the examples considered, (64) and (72) always invert a scalar, regardless of the autoregressive order. Matrices $\mathbf{F}$, $\mathbf{H}$ and $\mathbf{Q}$ have many zero elements (see example below), which simplifies the required matrix calculations.

Next, we consider an example of a fourth-order autoregression

$$\dot{h}_j(n)=\alpha_1\dot{h}_j(n-1)+\alpha_2\dot{h}_j(n-2)+\alpha_3\dot{h}_j(n-3)+ \\ +\alpha_4\dot{h}_j(n-4)+\dot{\xi}_j(n). \quad (99)$$

The matrices required for the Kalman filter to operate are determined by the expressions:

$$\mathbf{F}=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{pmatrix},\ \mathbf{H}=\left[E_s,0,0,0\right] \quad (100)$$

$$\mathbf{Q}_j=\begin{pmatrix}\sigma^2_{\xi,j} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\ \mathbf{R}=\sigma_u^2 \quad (101)$$

Matrices (100)-(101) contain many zero values, which eliminates some arithmetic operations and reduces the computational complexity of the algorithm. The estimation of the computational complexity of the channel coefficient estimation algorithm will be provided next, taking all information the above into account.

Prediction of the estimation of the channel coefficient is conducted according to the formula

$$\hat{\mathbf{h}}^-_{n,j}=\mathbf{F}\hat{\mathbf{h}}_{n-1,j}=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\hat{\dot{h}}_j(n-1)\\ \hat{\dot{h}}_j(n-2)\\ \hat{\dot{h}}_j(n-3)\\ \hat{\dot{h}}_j(n-4)\end{pmatrix}=\begin{pmatrix}\hat{\dot{h}}_j^-(n)\\ \hat{\dot{h}}_j^-(n-1)\\ \hat{\dot{h}}_j^-(n-2)\\ \hat{\dot{h}}_j^-(n-3)\end{pmatrix} \quad (102)$$

This requires only $2N_{ar}$ multiplications and $2\left(N_{ar}-1\right)$ additions to calculate the coefficient $\hat{\dot{h}}_j^-(n)$, taking into account the complexity of the coefficients and the real nature of the factors α_k, $k=1,\ldots,N_{ar}$. We do not count multiplications by units.

The prediction of the covariance (error) matrix is determined by the formula

$$\hat{\mathbf{P}}^-_{n,j}=\mathbf{F}\hat{\mathbf{P}}_{n-1,j}\mathbf{F}^T+\mathbf{Q}_j=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{pmatrix}\times \\ \times\hat{\mathbf{P}}_{n-1,j}\begin{pmatrix}\alpha_1 & 1 & 0 & 0\\ \alpha_2 & 0 & 1 & 0\\ \alpha_3 & 0 & 0 & 1\\ \alpha_4 & 0 & 0 & \end{pmatrix}+\begin{pmatrix}\sigma^2_{\xi,j} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

This requires N_{ar}^2 multiplications and $N_{ar}\left(N_{ar}-1\right)$ additions for multiplying $\mathbf{F}\hat{\mathbf{P}}_{n-1,j}$, and, then, N_{ar}^2 multiplications and $N_{ar}\left(N_{ar}-1\right)$ additions for multiplying by $\mathbf{F}^T$, and just one addition when summing with $\mathbf{Q}_j$. Finally, $2N_{ar}^2$ multiplications, $2N_{ar}\left(N_{ar}-1\right)+1$ additions are demanded. It should be mentioned, that the complexity of the product of matrices does not grow cubically with their size, as it is in the general, but quadratically.

We will consider the computational complexity of calculating the matrix of Kalman filter gain coefficients

$$\mathbf{K}_{n,j}=\hat{\mathbf{P}}^-_{n,j}\mathbf{H}^T\left(\mathbf{H}\ \hat{\mathbf{P}}^-_{n,j}\mathbf{H}^T+\mathbf{R}\ \right)^{-1}$$

in parts.

The product of $\hat{\mathbf{P}}^-_{n,j}\mathbf{H}^T$ requires only N_{ap} multiplications due to the form of $\mathbf{H}$ (see (100)). The product of $\mathbf{H}\ \hat{\mathbf{P}}^-_{n,j}\mathbf{H}^T$ is equal to $E_s^2\hat{P}^-_{n,j}(1,1)$, so only one multiplication is needed since square of signal energy E_s^2 can be calculated in advance. Thus, the expression in brackets before the inversion is a scalar, and its calculation requires only one multiplication and one addition. So, the computationally expensive matrix inversion is eliminated. Instead of it, a single scalar inversion is performed.

Prediction of the expected observations at the current step is calculated as

$$\hat{\mathbf{y}}^-_{n,j}=\mathbf{H}\hat{\mathbf{h}}^-_{n,j}$$

It demands 2 multiplications (taking into account that vectors are complex).

Refining the estimation using the formula

$$\hat{\mathbf{h}}_{n,j}=\hat{\mathbf{h}}^-_{n,j}+\mathbf{K}_{n,j}\left(\mathbf{y}_{n,j}-\hat{\mathbf{y}}^-_{n,j}\right)$$

requires $2N_{ar}$ multiplications and $2N_{ar}+2$ additions.

Also, the computational complexity of covariance (error) matrix of the channel coefficient estimation

$$\hat{\mathbf{P}}_{n,j} = \left(\mathbf{I} - \mathbf{K}_{n,j}\mathbf{H}\right)\hat{\mathbf{P}}^{-}_{n,j}$$

should be considered. The product of $\mathbf{K}_{n,j}\mathbf{H}$ requires N_{ap} multiplications. Calculation of the $\left(\mathbf{I} - \mathbf{K}_{n,j}\mathbf{H}\right)$ demands N_{ar} additions. The result of $\left(\mathbf{I} - \mathbf{K}_{n,j}\mathbf{H}\right)$ leads to the matrix

$$\left(\mathbf{I} - \mathbf{K}_{n,j}\mathbf{H}\right) = \begin{pmatrix} \beta_1 & 0 & 0 & 0 \\ \beta_2 & 1 & 0 & 0 \\ \beta_3 & 0 & 1 & 0 \\ \beta_4 & 0 & 0 & 1 \end{pmatrix} \quad (103)$$

where β_k, $k = 1,\dots,N_{ar}$ has values different from one or zero. The product of (103) and matrix $\hat{\mathbf{P}}^{-}_{n,j}$ requires N^2_{ar} multiplications and $N_{ar}\left(N_{ar} - 1\right)$ additions.

Finally, the total number of computational operations (multiplications R_{prod} and additions R_{sum}) with real numbers are

$$R_{prod} = 3N^2_{ar} + 5N_{ar} + 3, \quad (104)$$

$$R_{sum} = 3N^2_{ar} + 2N_{ar} + 2. \quad (105)$$

Thus, the computational complexity of the algorithm of the channel coefficient estimation is proportional to $3N^2_{ar}$ and grows quadratically with the autoregressive order, rather than cubically, as might be expected with matrix multiplication. The time-consuming matrix inversion operation is eliminated; instead, it is replaced by scalar inversion.

Figure 3 shows graphs of the number of arithmetic operations plotted using formulas (104) and (105). Figure 4 shows the increase in computational complexity versus the increase in the autoregressive order by one unit. For example, a value of ~1.6 for $N_{ar} = 4$ means that increasing the autoregressive order from 3 to 4 lead to the increase in computational complexity by a factor of ~1.6, i.e., by approximately 60%.

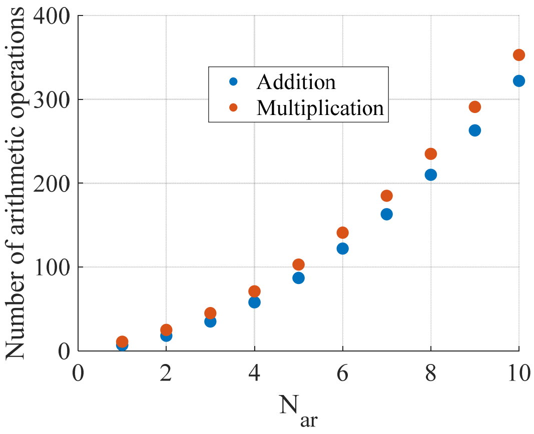


Fig. 3. Number of arithmetic operations

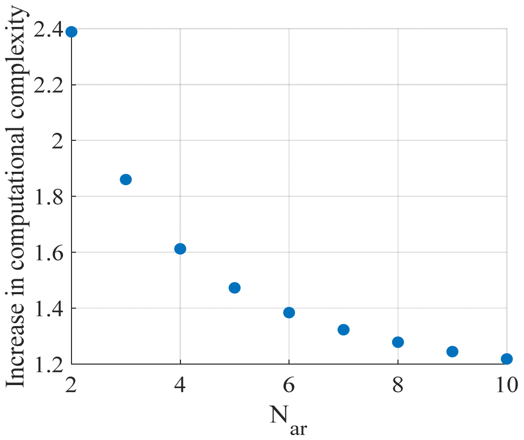


Fig. 4. The increase in computational complexity

Figures 5-7 shows graphs of the symbol error rate before the decoder when processing a radiogram which is used in the modem prototype [5]. It can be seen that estimating the channel coefficients using a Kalman filter based on data with third-order autoregression does not provide better noise immunity than estimating the channel coefficients using the maximum likelihood method based on the measurement channel.

Using higher-order autoregressions (4th and higher) provides better noise immunity than the ML estimator at low signal-to-noise ratios. However, 5th- and 6th-order autoregressions demonstrate nearly identical noise immunity.

As the SNR level increases, noise immunity decreases and becomes worse than for ML estimations. This stems from the factor that the gain from filtering decreases dramatically since the error in the resulting estimations caused by errors in decision making regarding data symbols.

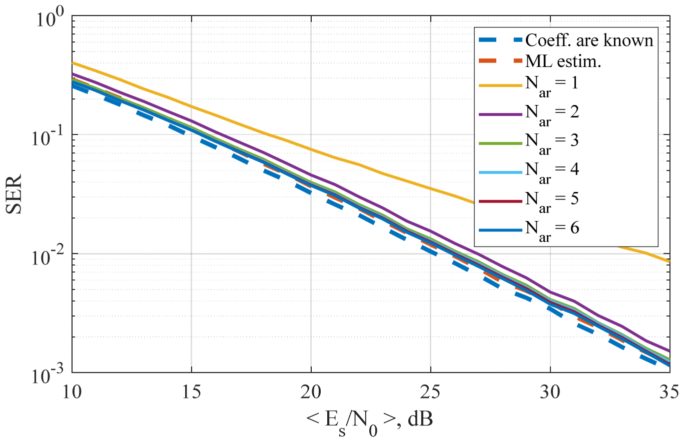


Fig. 5. Symbol error rate, Waterson channel coefficients are estimated by filters with different order of autoregression

The difference in SNR for identical noise immunity between 3rd- and 4th-order autoregressions at low SNRs is approximately 0.2 dB and 0.1 dB at high SNRs. The similar difference between 4th- and 6th-order autoregressions. It is less than 0.1 dB and approximately 0.25 dB.

The proportion of correctly received radiograms after decoding the code words of the NB-LDPC code versus the autoregression length N_{ap} is shown in Figure 8, and the estimation of the bit error rate is shown in Figure 8. The difference in SNR with the same noise immunity (the proportion of correctly received blocks is about 0.95, BER between 0.001 and 0.01) between the autoregressions of the 3rd and 4th orders is from 0.3 dB to 0.5 dB. Between the autoregressions of the 4th, 5th and 6th orders, the difference is insignificant.

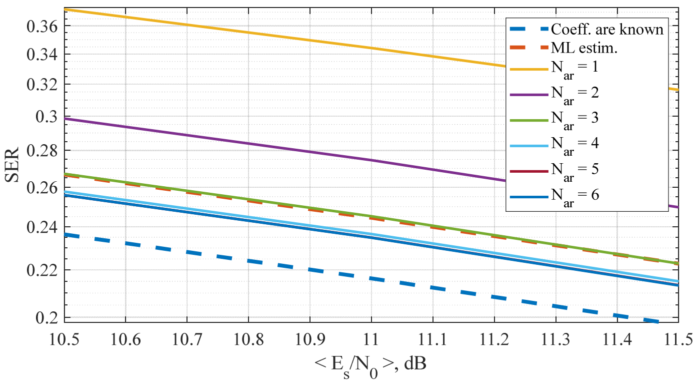


Fig. 6. Symbol error rate for low SNR, Waterson channel coefficients are estimated by filters with different order of autoregression

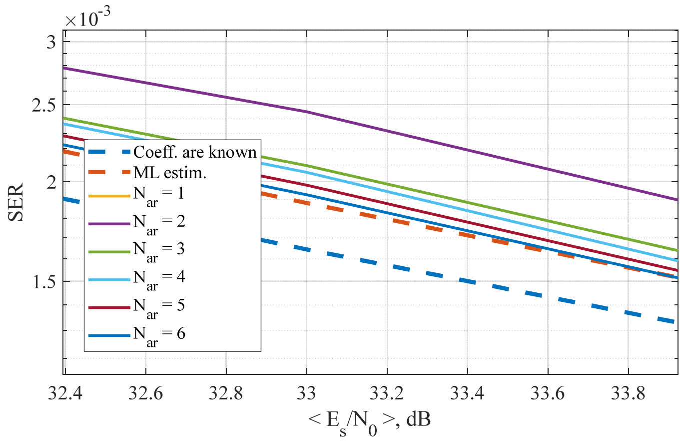


Fig. 7. Symbol error rate for high SNR, Waterson channel coefficients are estimated by filters with different order of autoregression

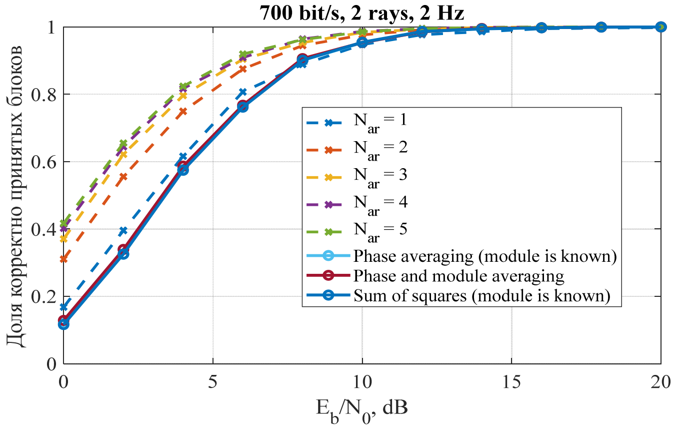


Fig. 8. The proportion of correctly received radiograms, different types of autoregression, the Doppler spread is 2.0 Hz

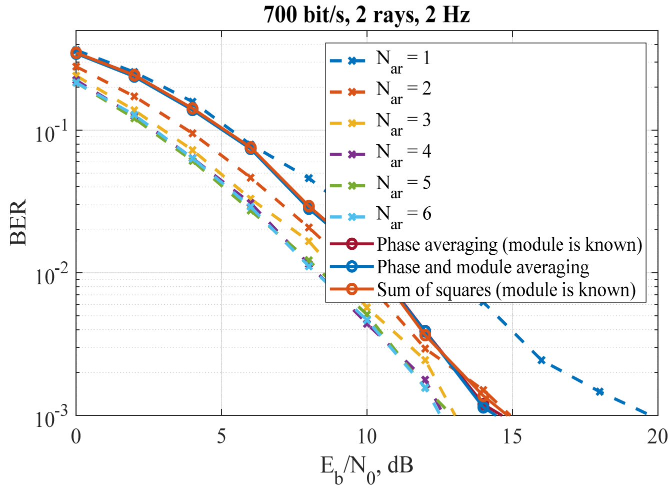


Fig. 9. Bit error rate, different types of autoregression, the Doppler spread is 2.0 Hz

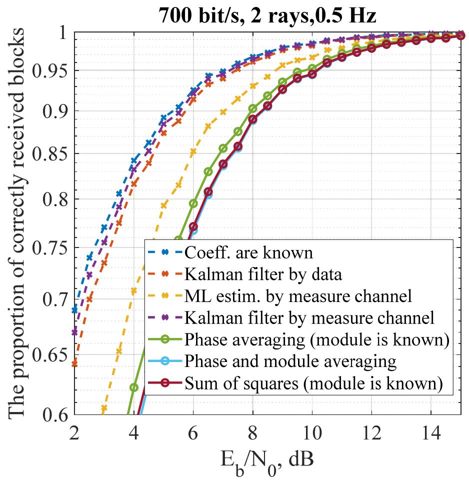
The obtained results indicate that increasing the autoregressive coefficient above the fourth order does not provide a significant increase in noise immunity, but increases computational complexity by 47%.

Therefore, for further research and application, we will use the Kalman filter with a fourth-order autoregression model.

Estimation of the noise immunity of algorithms for coherent processing of non-binary wideband signal-code structures

Figures 10-12 show the noise immunity curves for processing a radiogram implemented in the modem prototype [5] for two different Doppler spread (0.5 and 2.0 Hz) in a dual-beam ionospheric channel.

The indicators of the quality of noise immunity are the proportion of correctly received radiograms (code blocks), the bit error rate, and the symbol error rate.



a)

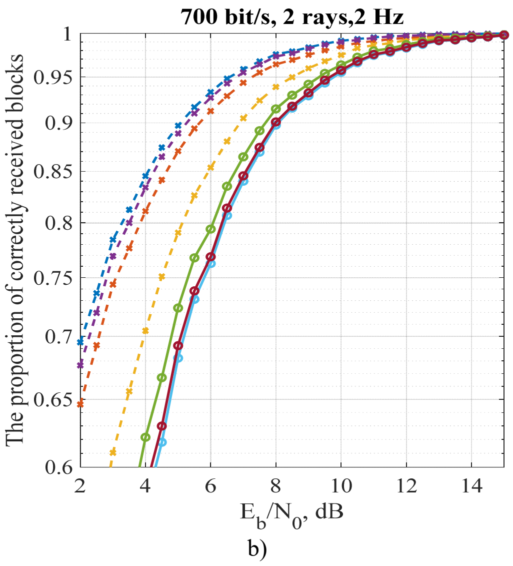


Fig. 10. The proportion of correctly received blocks, Doppler spread is 0.5 Hz (a) and 2.0 Hz (b)

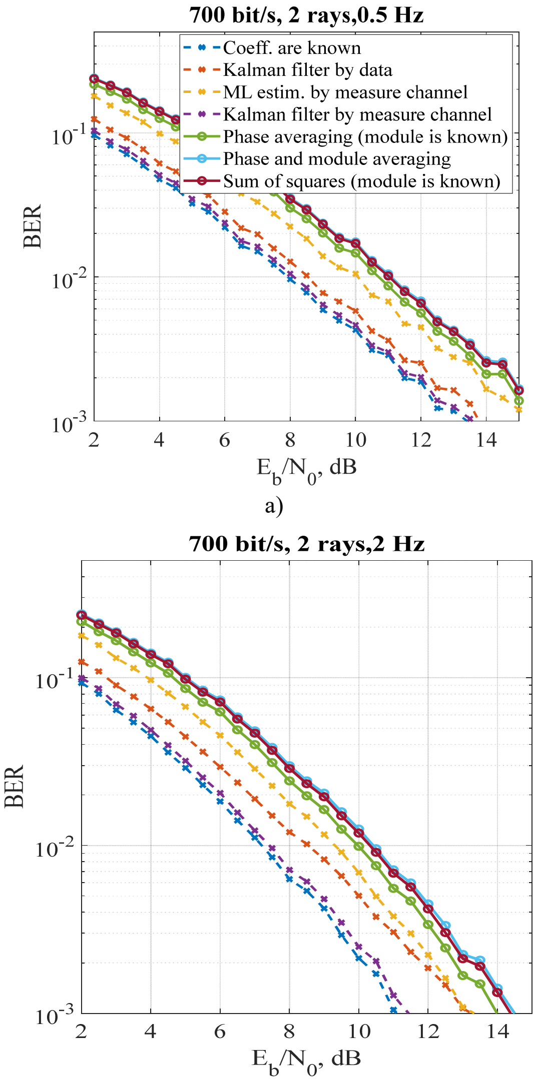


Fig. 11. Bit error rate, Doppler spread is 0.5 Hz (a) and 2.0 Hz (b)

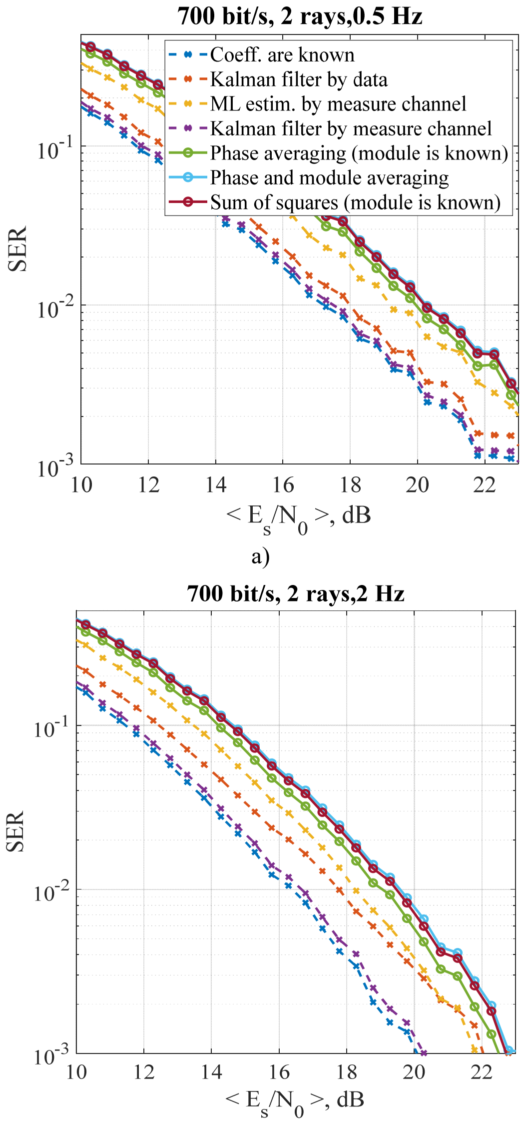


Fig. 12. Symbol error rate, Doppler spread is 0.5 Hz (a) and 2.0 Hz (b)

Radiogram processing has been performed using non-coherent algorithms, generating posterior probabilities taking into account (38), (43), (46), and the coherent algorithm. For the coherent algorithm, various values of channel coefficients were used: known values, values obtained via the measurement channel, values obtained via the measurement channel and processed by the Kalman filter, and values obtained by processing radiogram samples by the Kalman filter (based on the preamble and data).

A comparison of the noise immunity curves shows that the coherent processing algorithm with Kalman filtering on the data provides an energy gain of about 2.4 dB compared to the quadratic addition algorithm, which is implemented in the modem prototype [5], at a level of the proportion of correctly received blocks of 0.98 and a Doppler spread is 0.5 Hz. A similar benefit is observed if we analyze the graphs of the bit and symbol error rates (if the same SNR is taken as the starting point for the coherent algorithm as for the proportion of correctly received blocks). For faster fading (Doppler spread is 2.0 Hz), the energy gain is 1.7 dB according to the graphs demonstrating the proportion of correctly received blocks and 1.5 dB according to the graphs of the error rates.

Conclusion

The article considers algorithms for processing wideband non-binary signal-code structures under conditions of partial or complete a priori uncertainty regarding the channel coefficients for each multipath component of the received signal.

Expressions are obtained for calculating the posterior probabilities for each variant of a non-binary symbol, taking into account multipath signal propagation under the following conditions:

– precise knowledge of the channel coefficients for each multipath component;

– averaging over the initial phase of the multipath components, assuming their uniform distribution;

– averaging over the phase and absolute value of the channel coefficients, assuming a Rayleigh distribution of absolute values;

– averaging only over the absolute value of the channel coefficients, given a known initial phase;

– quadratic addition of the multipath components with known absolute values of the channel coefficients (the variant implemented in the modem prototype [5]).

The algorithm has been developed for coherent processing of radiograms transmitted by wideband non-binary signal-code structures. This algorithm uses estimations of channel coefficients for each multipath component provided by the Kalman filter that works on radiogram preambles and transmitted data symbols (with corresponding decision-making).

To conduct comparative noise immunity assessments and validate key aspects of the algorithm, coherent reception options are also considered, in which channel coefficients are estimated using the maximum likelihood method on a separate available measurement channel and additional processing of these estimations by the Kalman filter. In real applications, a separate measurement channel is not available.

The example of radiograms from a modem model has shown that the first-order autoregression is insufficient for receiving radiograms with noise immunity comparable to the levels achievable using maximum likelihood estimations of channel coefficients.

To improve the noise immunity, a higher-order autoregressive model has been constructed based on the Waterson ionospheric channel model by solving the Yule-Walker equation. Evaluation of the noise immunity and algorithm complexity for optimal filtering with increasing autoregressive order under dual-beam ionospheric channel conditions have shown that a fourth-order autoregressive model is sufficient. Increasing the autoregressive order beyond fourth does not significantly improve noise immunity, but increases computational complexity by 47%.

The noise immunity estimations for the developed algorithm have been obtained in the form of curves representing the proportion of correctly received radiograms and the symbol and bit error rates versus the average signal-to-noise ratio in a two-beam ionospheric channel. A comparison of the noise immunity shows that the coherent processing algorithm with Kalman filtering utilizing data provides the energy gain of about 2.4 dB compared to the quadratic addition algorithm, which is implemented in the modem prototype at a level of the proportion of correctly received blocks equal to 0.98 and a Doppler spread is 0.5 Hz. The similar level of the benefit is observed in the analysis of the graphs of the bit and symbol error rates (if the same SNR as for the proportion of correctly received blocks is chosen as the starting point for the coherent algorithm). For faster fading (Doppler spread is 2.0 Hz), the energy gain is 1.7 dB according to the graphs of the proportion of correctly received blocks and 1.5 dB according to the graphs of the error rates.

Thus, the developed algorithm provides an energy gain of up to 2.4 dB compared to known, implemented analogs.

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АЛГОРИТМ КОГЕРЕНТНОЙ ОБРАБОТКИ ШИРОКОПОЛОСНЫХ НЕДВОИЧНЫХ СИГНАЛЬНО-КОДОВЫХ КОНСТРУКЦИЙ ДЛЯ ПЕРЕДАЧИ РЕЧИ В ДЕКАМЕТРОВОМ РАДИОКАНАЛЕ

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Аннотация

В настоящей статье приводятся результаты разработки алгоритма когерентной обработки широкополосных недвоичных ортогональных сигнально-кодовых конструкций для передачи речи в декаметровом радиоканале. Приводится структура радиограммы, включающая в себя символы преамбулы и символы данных с учетом ограничений на длину сообщения при передаче речи в реальном масштабе времени. В качестве сигналов, используется ансамбль ортогональных широкополосных фазоманипулированных сигналов. Используемый помехоустойчивый код - недвоичный код с низкой плотностью проверок на четность, размерность поля Галуа которого согласована с числом сигналов в упомянутом ансамбле. Декодирование проводится алгоритмом распространения доверия, который предполагает вычисление апостериорных вероятностей по каждому варианту возможного переданного символа с учетом наблюдений. В статье приведены аналитические выражения для вычисления указанных вероятностей с учетом многолучевого распространения сигнала, при условии разделения лучей без их взаимного влияния, а также с учетом различной априорной неопределенности относительно комплексных коэффициентов передачи канала для каждого луча. Предполагается три варианта априорной информации: полностью известные комплексные коэффициенты передачи канала, неизвестный фазовый сдвиг коэффициентов передачи канала или неизвестный фазовый сдвиг и уровень коэффициентов передачи канала. Дополнительно рассматривается четвертый, широко используемых вариант квадратичного сложения лучей. При когерентной обработке комплексные коэффициенты передачи канала предполагаются известными или измеренными каким-либо образом. В статье рассматривается несколько вариантов измерения комплексных коэффициентов передачи канала: по отдельному каналу измерений методом максимального правдоподобия и с использованием фильтра Калмана, который в одном варианте работает по данным отдельного канала измерения, в другом варианте - по символам преамбулы и по символам данных радиограммы с обратной связью по решениям относительно символов данных. С использованием модели Вотерсона ионосферного канала в статье обосновывается порядок авторегрессии фильтра Калмана с позиции сложности алгоритма и достижимой помехоустойчивости. Приводятся кривые помехоустойчивости алгоритмом когерентной обработки с использованием фильтра Калмана и алгоритмов некогерентной обработки. Оценивается соответствующей энергетический выигрыш.

Ключевые слова: *декаметровый диапазон, недвоичные ортогональные сигнально-кодовые конструкции, широкополосный сигнал, фильтр Калмана, апостериорные вероятности*

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