ANALYSIS OF SELECTION CRITERIA FOR VECTOR CHANNELS OR ORTHOGONAL PRECODING MATRICES IN COMMUNICATION SYSTEMS WITH MULTIPLEXING

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Modern multiuser communication systems use various technologies for multiplexing symbol streams over multiple parallel channels (OFDM, MIMO). Channels with different types of multiplexing generally refer to vector channels. There is a problem of selecting the best vector channel for the user from the available set. When it is not possible to select a vector channel, the performance may be improved by using a precoding matrix that reallocates the streams across the subchannels. In this case, there is also a problem of selecting the best precoding matrix. Not always the traditional selection criteria used for scalar channels, such as, for example, maximum average SNR or maximum throughput capacity, provide the best choice in terms of data transmission quality. Several variants of known criteria for vector channel selection or orthogonal precoding are considered, and new empirical criteria are proposed. The mathematical modeling method analyzed the efficiency of using different criteria for the OFDMA communication system when selecting a set of subcarriers, as well as when selecting an orthogonal precoding matrix. Efficiency of using different criteria for an OFDMA communication system is analyzed via simulation, when selecting a set of subcarriers, as well as when selecting an orthogonal precoding matrix. The results of the analysis show that empirical criteria often give better results compared to traditional ones.

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Introduction

4G – 5G communications are based on multiplexing channels when modulated symbols are transmitted via parallel streams. OFDM [1-5] works with multiple subcarriers where incoming bit stream divided into parallel streams.

The other example for parallel streams is MIMO communication system [2, 6-12] with multiple transmitting and receiving antennas where bit stream is being divided into parallel communication channels.

Those multiplexing operation might be represented as conversion of scalar bit stream into vector stream and further math manipulations may imply vector – matrix operations. Vector-matrix presentation of communication systems is common for MIMO systems with multiple transmitting and receiving antennas.

This paper considers the communication systems which operates vector channels [13-23] though they are might be channels in terms of MIMO but we may use "vector channel" as the term which may cover wider range than MIMO itself.

Communication systems which use parallel channels like OFDM or MIMO face challenges of non-optimal channels utilization per one user. For example, in ODFM the whole subcarriers set is divided into subsets which are assigned for users. For MIMO systems the number of transmitters might be less than antennas. The problem is that we need to find optimal algorithms to allocate resources (subsets or MIMO channels) at multi-user systems.

The issue of granting resources for users raises the problem of feedback loops to provide information about communication channel at transmission part. The optimal capacity of feedback channels is very important for the communication systems [24-29].

Replacing the vector channel (which indicates the communication channel) by preset number (indicator) can be used to reduce the amount of data transmitted via feedback loop. The approach also may use for precoding which implemented as vector-matrix multiplication and we need to choose the appropriate matrix [19, 23, 28, 29-34].

Complete information about communication channels should be available at receiving side of communication system to improve an efficiency. In that situation precoding matrix elements may have different values, and we need more capacity for feedback channel to transmit it. The alternative way is finite variants of those precoding matrices or codebooks at transmitting side of the system and then transmit only preset indicator of chosen variant via feedback loop.

This approach is considered at [34-36] and applied for modern networks based on LTE and LTE Advanced [37-40] in addition it can be applied for new communication systems which based on NOMA (Non Orthogonal Multiple Access) [41-43].

The selection among available vector channels is a problem which needs to be solved in all the communication systems we have mentioned. This selection should be determined by selection criteria but not every criterion will define the efficiency of communication system (like noise immunity). Our paper gives some analysis of several selection criteria and then we explored relationship between efficiency of communication system with "vector channels" and various selection criteria.

1 System model

Let consider communication systems with vector channels. Multiplexing is used for data transmission via vector channels [1, 4] when stream of complex modulated symbols s_i, $i=1,2,...$ are splitted into number of vectors $\mathbf{x}_n = \left[x_n^{(1)} \quad x_n^{(2)} \quad \cdots \quad x_n^{(M)} \right]^T$ – $(M \times 1)$ dimensioned where m-th element of n-th vector is $x_n^{(m)} = s_{(n-1)N+m}$, $m=\overline{1,M}$, $n=1,2,...$.

Every modulated symbol holds zero mean $E\{s_i\}=0$ and unit power $E\left\{|s_i|^2\right\}=1$. System model for communication system with vector channels might be expressed as [4, 6, 7]:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\eta} \tag{1}$$

where $\mathbf{y} = \left[y^{(1)} \quad y^{(2)} \quad \cdots \quad y^{(M)} \right]^T$ – is $(M \times 1)$ – dimensioned vector of received signal; $\mathbf{H}$ is – $(M \times M)$ - dimensioned channel matrix which consists of random complex transmission coefficients; $\boldsymbol{\eta}$ – is $(M \times 1)$ - dimensioned Gaussian noise vector with covariance matrix $\mathbf{R}_\eta = E\left\{\boldsymbol{\eta}\boldsymbol{\eta}^H\right\}$, which are diagonal one with equal diagonal elements $\mathbf{R}_\eta = \frac{1}{\rho}\mathbf{I}_M$, where $\mathbf{I}_{N_{rx}}$ – is identity matrix with dimension $(M \times M)$, $\rho = \frac{P_s}{2\sigma_\eta^2}$ – is mean SNR at receiving antenna, P_s – is mean signal power at receiving antenna, σ_η^2 – is Gaussian noise variance for both real and imaginary parts. We suppose that channel matrix $\mathbf{H}$ and noise variance σ_η^2 are known at receiving part of the system and in addition we restrict ourselves to the situation when channel matrix is square.

There are different examples of vector channels such as MIMO with M transmitting and receiving antennas [6-7] for that type the channel matrix is fully filled.

Spatial multiplexing system model [44-45] is a type of MIMO system. MIMO systems with one receiving antenna have remained vector channels system with square channel matrix. That matrix (EVCM) is a squared sparse matrix of equivalent virtual channel [46].

The other example of vector channels for communication system with multiplexing is multiuser communication systems based on OFDM [1-5]. The system model might be expressed by (1) after some transformation (cyclic prefix removal, Fourier transform). In this case, the channel matrix is diagonal and has a size defined by number of subcarriers per one user.

Communication system based on HSDPA is another example with multiplexing when set of code sequences used for the process [47]. The system model is also might be expressed by (1) after correlation processing. The channel matrix is defined by channel physical properties and cross-correlation between the code sequences.

For further analysis we may use communication systems with number of vector channels, and we suppose that there are V different matrices of vector channels $\mathbf{H}^{(i)}$, $i=\overline{1,V}$ and all of them known at the receiving part.

In that case the system model is multivariate model.

$$\mathbf{y} = \mathbf{H}^{(i)}\mathbf{x} + \boldsymbol{\eta},\ i = \overline{1,V}, \tag{2}$$

where i – is might be preset and known at both transmitting and receiving sides of system or might be selected by subscriber in terms of special requirements and then back to transmission side via feedback loop.

For OFDM communication systems that i set would be the number of subcarriers divided by blocks which may be assigned to different users. The system may allow users to select one of several blocks if they are not occupied. The choice is to select the best matrix and pass the information about the choice to transmission part of the system. Mathematically, that approach is described as follows.

There is a set of vector channel matrices $\mathbf{H}^{(i)}$, $i = \overline{1,V}$ and then i_{opt} may be brute forced, where

$$i_{opt} = \arg_{i=\overline{1,V}} \Lambda\left\{\mathbf{H}^{(i)}, \sigma_\eta^2\right\}. \tag{3}$$

Mathematical operator $\Lambda(\bullet)$ declares chosen optimum criteria for the number of vector channels.

The system model with optimal number is given below:

$$\mathbf{y} = \mathbf{H}^{(i_{opt})}\mathbf{x} + \boldsymbol{\eta} \tag{4}$$

For communications systems with precoding the system model is

$$\mathbf{y} = \mathbf{H}\mathbf{F}^{(i)}\mathbf{x} + \boldsymbol{\eta}, \tag{5}$$

where $\mathbf{F}^{(i)}$ is precoding matrix belonging to the set of V matrices with different properties. The task is to select the optimal precoding matrix.

Using an equivalent channel matrix the precoding system model (5) could be presented as model (2) view

$$\mathbf{y} = \mathbf{H}_{eq}^{(i)}\mathbf{x} + \boldsymbol{\eta}, \tag{6}$$

where $\mathbf{H}_{eq}^{(i)} \triangleq \mathbf{H}\mathbf{F}^{(i)}$.

The selection for optimal matrix will be defined by (3). It means that the model for vector channels optimal selection and for precoding matrix is described by same expressions and applying the same criteria. General model with vector channel selection is given by the

Fig. **1** and it has coding and modulation blocks in the transmission part. Symbol estimation, demodulation and decoding blocks are un the receiving part of the system.

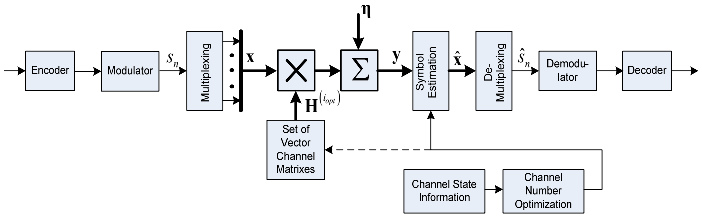

Fig. 1. General model with vector channel selection

Our system model suggests that the full information about all the vector channels is at receiving side. This is achieved by pilot symbols or pilot sequences for channel estimation algorithms.

The information is needed for vector channel number identification. This assigned number should be passed to transmitting part of the system via feedback loop and then using at symbol estimation block.

Selection optimization criteria

There are different criteria for vector channel selection with various target functions and optimization algorithms. It is possible to use exhaustive search methods for a small number of channels. Maximization of SNR is quite common [48] for antenna diversity system for scalar channel selection.

The remaining quality characteristics of communication systems including capacity, errors probability etc. seem to be linked to SNR. SNR for vector channels is random vector with some distribution function and some parameters. Vector channel realization might be considered as a sample with different parameters values. The most common parameter for vector channel realization might be mean SNR.

Maximum mean SNR often occurs for practice [49-50]. For system model (2) SNR vector consist of SNR values per every channel as it is given below:

$$\rho_m^{(i)} = \sum_{k=1}^{M} \frac{\left|h_{km}^{(i)}\right|^2}{2\sigma_{\eta,m}^2}, \tag{7}$$

where $h_{km}^{(i)}$ – (k,m) – is the element of channel matrix $\mathbf{H}^{(i)}$, $2\sigma_{\eta,m}^2$ – noise variance of m-th channel output.

Thus, the first considered selection criteria is maximum of mean SNR which applied to vector channel (channel matrix) number selection. The goal function for maximum mean SNR is:

$$\Lambda_{\max\overline{SNR}} = \max\left(\frac{1}{M}\sum_{m=1}^{M}\rho_m^{(i)}\right). \tag{8}$$

If the noise variances for all channel vector elements are same $\sigma_{\eta,m}^2 = \sigma_\eta^2$, $m = \overline{1,M}$, then our maximum mean SNR criterion leads to another criteria which is maximum of Frobenius norm for channel matrix:

$$\Lambda_{\max\|F\|} = \max\left(\sum_{m=1}^{M}\sum_{k=1}^{M}\left|h_{mk}^{(i)}\right|^{(2)}\right) = \max\left(\sqrt{\sum_{m=1}^{M}\sum_{k=1}^{M}\left|h_{mk}^{(i)}\right|^{(2)}}\right). \tag{9}$$

Frobenius norm is very popular because of computational simplicity and very often used for antenna selection in MIMO systems [51-53]. One of the criteria varieties for maximum mean SNR is maximum minimum SNR value which means that we select the variant when minimal SNR have reached the maximum value.

Optimal signal processing for vector channel communication systems may be implemented as exhaustive search algorithm within the whole set of combinations. That approach might be applied for analytical error estimation only for a narrow class of models. Therefore, preliminary linear estimation of modulated symbols vector $\mathbf{X}$ based on MMSE receiver [20, 23, 54-62] could be proposed. Estimation quality is enough for linear estimation algorithms.

For MMSE-based algorithm we have:

$$\hat{\mathbf{x}}_{MMSE}^{(i)} = \left(\mathbf{H}^{(i)\prime}\mathbf{H}^{(i)} + 2\sigma_\eta^2\mathbf{I}_M\right)^{-1}\mathbf{H}^{(i)\prime}, \qquad (10)$$
$$\mathbf{R}_{MMSE}^{(i)} = 2\sigma_\eta^2\left(\mathbf{H}^{(i)\prime}\mathbf{H}^{(i)} + 2\sigma_\eta^2\mathbf{I}_M\right)^{-1}$$

where $\hat{\mathbf{x}}_{MMSE}^{(i)}$ – is MMSE-based estimations vector of modulated symbols for *i*-th vector channel, $\mathbf{R}_{MMSE}^{(i)}$ – is covariance matrix of MMSE – based algorithms estimation errors, $\mathbf{A}'$ – is complex Hermitian conjugate matrix for initial matrix $\mathbf{A}$. Variances of estimation errors $v_{m,MMSE}^{(i)} = R_{mm,MMSE}^{(i)}$ might be indicators of channel quality. The variances vector $\mathbf{v}_{MMSE}^{(i)}$ is random and depends on channel parameters what means to be used for best selection variant.

For example, it might use those variances for probability estimations of bit or symbol rates. For QPSK modulation bit-error-rate per one symbol expressed by [58]:

$$\mathrm{BER}_m^{(i)} = Q\left(\frac{1}{\sqrt{v_{m,MMSE}^{(i)}}}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{2v_{m,MMSE}^{(i)}}}\right), \qquad (11)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}dt$, $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$.

In this case a minimum error probability can be used:

$$\Lambda_{\min\overline{\mathrm{BER}}} = \min\left(\frac{1}{M}\sum_{m=1}^{M}Q\left(\frac{1}{\sqrt{v_{m,MMSE}^{(i)}}}\right)\right). \qquad (12)$$

It is possible to use rough estimation for error probability error which is not associated with modulation type:

$$\widehat{\mathrm{BER}}_m^{(i)} = \frac{1}{2}\exp\left(-\frac{1}{2v_{m,MMSE}^{(i)}}\right). \qquad (13)$$

The relevant criteria for this indicator s:

$$\Lambda_{\min\widehat{\mathrm{BER}}} = \min\left(\frac{1}{2M}\sum_{m=1}^{M}\exp\left(-\frac{1}{2v_{m,MMSE}^{(i)}}\right)\right). \qquad (14)$$

It should be noted that the variances of estimation might be used as criteria itself. Mean variance (the same as mean SNR) might be the indicator for quality of communication systems. The mean variance criteria is:

$$\Lambda_{\min\overline{\mathbf{v}_{MMSE}}} = \min\left(\frac{1}{M}\sum_{m=1}^{M}v_{m,MMSE}^{(i)}\right) \qquad (15)$$

This criteria is inappropriate for orthogonal precoding [30] due to that transformation for symbols vector does not affect covariance matrix trace modification and hence mean variances do not change. That is the reason why proposed empirical criteria for precoding matrix selection based on minimal value of maximum variances:

$$\Lambda_{\min\max\mathbf{v}_{MMSE}} = \min\left(\max_{m=\overline{1,M}}\left(v_{m,MMSE}^{(i)}\right)\right). \qquad (16)$$

This criteria has worked well for MIMO communications systems with extended orthogonal precoding [34]. Channel capacity is often used for analysis, comparison and optimization for different signal code constructions and parameters. Channel capacity for MIMO Gaussian channel has described at [8, 13] and might be applied for system models (2) and (6):

$$C^{(i)} = \log_2\det\left(\mathbf{I}_M + \frac{1}{2\sigma_\eta^2}\mathbf{H}^{(i)\prime}\mathbf{H}^{(i)}\right). \qquad (17)$$

Optimal selection criteria for that case expressed as:

$$\Lambda_{\max C} = \max\left(\log_2\det\left(\mathbf{I}_M + \frac{1}{2\sigma_\eta^2}\mathbf{H}^{(i)\prime}\mathbf{H}^{(i)}\right)\right). \qquad (18)$$

Another empirical criteria is opposite to maximum SNR mean value (8) (or maximum of Frobenius norm of channel matrix (9)):

$$\Lambda_{\min\|SNR^{-1}\|} = \min\left(\sum_{k=1}^{M}\left(\sum_{m=1}^{M}\left|h_{mk}^{(i)}\right|^2\right)^{-1}\right). \qquad (19)$$

Calculation of mean SNR with same variances of noise at every channel linked with square of Euclidean norm (or Frobenius norm) for channel matrix by following expression:

$$\left(\|\mathbf{H}\|_F\right)^2 = \mathrm{trace}\left(\mathbf{H}'\mathbf{H}\right) = \mathrm{trace}\left(\mathrm{diag}\left(\mathbf{H}'\mathbf{H}\right)\right), \qquad (20)$$

where $\mathrm{trace}(\mathbf{A})$ – is matrix trace of $\mathbf{A}$, $\mathrm{diag}(\mathbf{A})$ – is formation operator of diagonal matrix consisting of diagonal elements of $\mathbf{A}$.

Combining (19) and (20) we may obtain:

$$\left(\sum_{k=1}^{M}\left(\sum_{m=1}^{M}\left|h_{mk}^{(i)}\right|^2\right)^{-1}\right) = \mathrm{trace}\left(\mathrm{diag}\left(\mathbf{H}'\mathbf{H}\right)^{-1}\right). \qquad (21)$$

We may see that the first step is calculating the sum of square modulus for each column $\sum_{m=1}^{M}\left|h_{mk}^{(i)}\right|^2$, second is calculating the sum of inverses for the elements. The physical idea for the criteria is that it considers the "worst" symbols. The square modulus for those symbols is small and the demodulation quality would be worse than for the others. Therefore, we may choose the variant of vector channel where the weight of those "worst" symbols is minimum. The criteria could be represented via norms of channel matrix column:

$$\Lambda_{\min\overline{\|\mathbf{h}\|^{-2}}} = \min\left(\frac{1}{M}\sum_{k=1}^{M}\left\|\mathbf{h}_k^{(i)}\right\|^{-2}\right), \qquad (22)$$

where $\|\mathbf{x}\| = \sqrt{\mathbf{x}'\mathbf{x}}$ – is vector norm of $\mathbf{x}$, $\mathbf{h}_k^{(i)}$ – is *k*-th column of matrix $\mathbf{H}^{(i)}$.

We may call this selection criteria as the minimum mean of inverse square norms of matrix channel columns. Thereafter a study of selection criteria will take place and simulation results for different conditions to confirm their efficiency.

Simulations and comparison of different selection criteria

Criteria described above are approximate and do not work as the absolute quality indicators for data transmission, especially for communication systems with error correction coding when the quality indicator is FER. That is the reason to provide simulation (see Fig. 1) for three following types of communication system:

1. Communication system with diagonal vector channel selection. Selection has been made from the whole set of vectors with uncorrelated fading at each channel. Users can select any vector channel of all V-vectors available. The simulation results demonstrate potential an efficiency of the proposed criteria.
2. OFDMA-based communication system with distributed arrangement of N_{sub} dedicated subcarriers per one user. There are V sets of available subcarriers and the user may select the best option. TDL-C [59] channel model is used for simulation.
3. OFDMA-based communication system with distributed arrangement of N_{sub} allocated subcarriers per user but the user can't select the best option as at type 2. These allocated subcarriers are not linked to channel environment. Orthogonal precoding with V set of orthogonal matrices (code books) uses for improving of reception quality. There are k basis orthogonal matrices used for orthogonal matrices set generation. The other orthogonal matrices are formed as all possible products for basis orthogonal matrices. The whole set of matrices is $V = 2^k$ including unitary matrix. TDL-C channel model is used for simulation as well.

Our simulation results will cover both vector channel selection and orthogonal precoding.

Communication system parameters and simulation conditions are at Table 1.

Table 1

Communication system parameters and simulation conditions

Coding	Turbo code (LTE Toolbox)
Code rate	3/4
Modulation type	QPSK
	16QAM
Signal structure	LTE radio interface
Number of OFDMA subcarriers	1024
Number of subcarriers per user	16
User subcarrier location	Distributed
Number of selection options	16
Propagation channel environment	Uncorrelated over subcarriers
	TDL-C
Demodulation	MMSE with soft QAM demodulator from LTE Toolbox
Decoding	LTE Toolbox

Selection criteria chosen for comparison:

1. Without selection;
2. Maximum of mean SNR (8) ('max Av. SNR');
3. Maximum of channel capacity (18) ('max Capacity');
4. Minimum of mean BER after MMSE detection (14);
5. Minimum of mean covariance for MMSE estimation error (15) ('min Av. Var_{MMSE}');
6. Minimum of maximum variance of MMSE estimation error (16) ('min max Var_{MMSE}');
7. Minimal sum (or mean) of inverse values of square norms for channel matrix columns (22) ('min Av. $\|\mathbf{h}_k\|^{-2}$');

Fig. 2 is plot for FER vs. SNR per bit for different criteria for OFDMA communication system with QPSK modulation and uncorrelated fading at the channels (subcarriers) (type 1 – with subcarrier selection). Fig. 3 is the same plot for TDL-C (type 2).

Fig. 4 is plot for OFDMA communication system without subcarriers selection but with orthogonal precoding (type 3).

For better presentation of criteria efficiency at Table 2 we have threshold SNR values FER = 0.01 и 0.001 for all the simulations and all criteria and values at the table are colored (red color – means biggest value, green color – means smallest value). There are the best SNR marked (minimum for every SNR column) and power gain in comparison with communication system without selection.

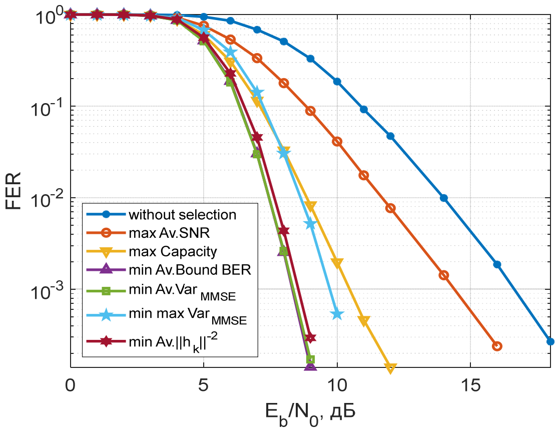

Fig. 2. FER for uncorrelated fading channel. QPSK modulation. (Type 1)

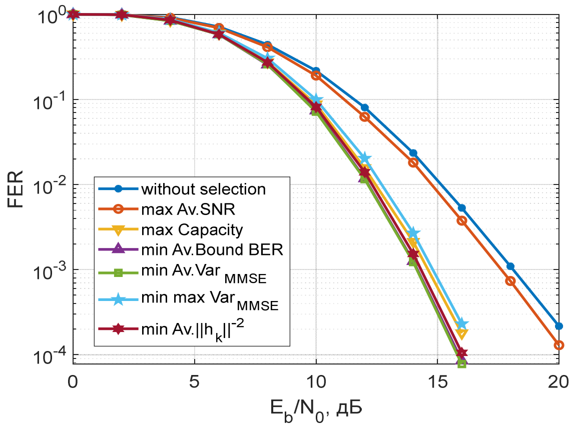

Fig. 3. TDL-C channel. QPSK modulation. (Type 2)

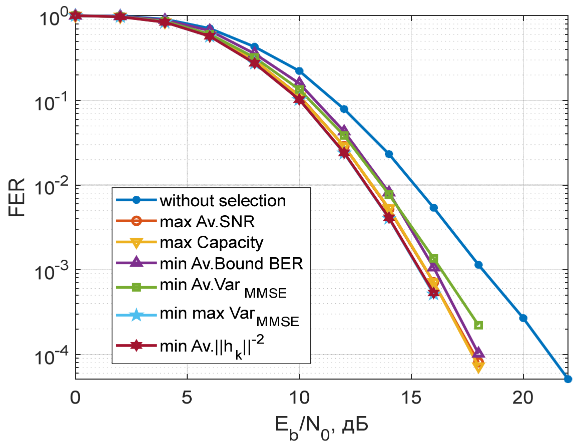

Fig. 4. TDL-C channel with orthogonal precoding matrix selection. QPSK modulation. (Type 3)

Table 2

Threshold SNR values for different criteria and fixed FER values. QPSK modulation

Criteria		Eb/N0, dB (FER=0.01)			Eb/N0, dB (FER=0.001)		
N	Type	Ver. 1	Ver. 2	Ver. 3	Ver. 1	Ver. 2	Ver. 3
1	without selection	13,991	15,172	15,181	16,676	18,111	18,182
2	max Av.SNR	11,682	14,791	13,272	14,403	17,63	15,671
3	max Capacity	8,865	12,469	13,295	10,459	14,655	15,685
4	min Av.Bound BER	7,47	12,158	13,781	8,338	14,185	16,056
5	min Av.Var$_{MMSE}$	7,469	12,136	13,693	8,364	14,159	16,346
6	min max Var$_{MMSE}$	8,654	12,742	13,016	9,758	14,852	15,393
7	min Av. $\lVert \mathbf{h}_k \rVert^{-2}$	7,667	12,322	13,031	8,563	14,341	15,418
Minimum		7,469	12,136	13,016	8,338	14,159	15,393
Gain		6,522	3,036	2,165	8,338	3,952	2,789

Fig. 5-7 and Table 3 demonstrates the same as figures before with the same simulation conditions but for 16 QAM modulation.

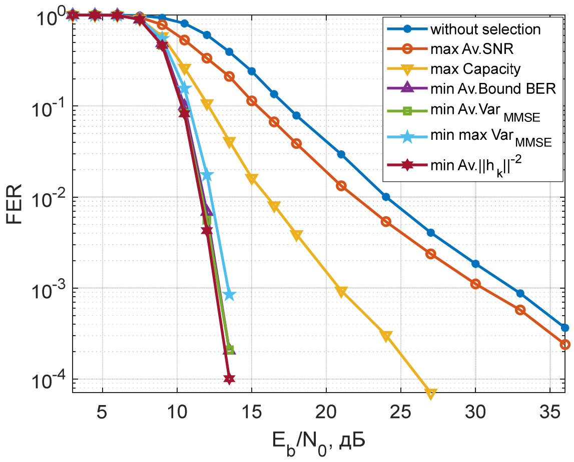

Fig. 5. Uncorrelated fading channel. 16QAM modulation (Type 1)

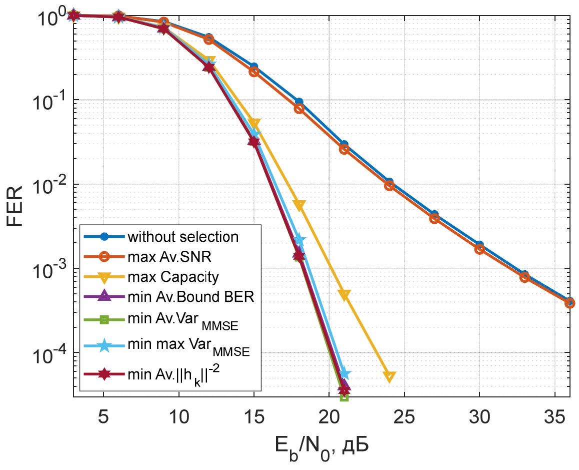

Fig. 6. TDL-C channel. 16QAM modulation (Type 2)

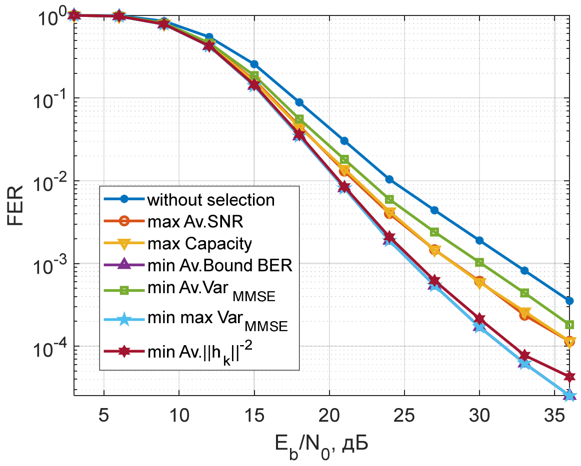

Fig. 7. TDL-C channel with orthogonal precoding matrix selection. QPSK modulation (Type 3)

Table 3

Threshold SNR values for different criteria and fixed FER values. 16QAM modulation.

Criteria		Eb/N0, dB (FER=0.01)			Eb/N0, dB (FER=0.001)		
N	Type	Ver. 1	Ver. 2	Ver. 3	Ver. 1	Ver. 2	Ver. 3
1	without selection	24,013	24,2	24,124	32,479	32,385	32,297
2	max Av.SNR	21,899	23,859	21,658	30,451	32,006	28,297
3	max Capacity	16,038	17,29	21,828	20,846	20,136	28,226
4	min Av.Bound BER	11,819	16,235	20,576	12,88	18,358	25,488
5	min Av.Var MMSE	11,704	16,189	22,568	12,822	18,257	30,114
6	min max Var MMSE	12,31	16,511	20,576	13,438	18,691	25,488
7	min Av. $\lVert \mathbf{h} \rVert^{-2}$	11,611	16,184	20,658	12,622	18,286	25,782
Minimum		11,611	16,184	20,576	12,622	18,257	25,488
Gain		12,402	8,016	3,548	19,857	14,128	6,809

The obtained simulation results allow to make the following conclusions:

1. There is no significant difference between criteria of minimum of mean estimation error (14) and minimum of mean inverse square norm (22). They both provide the best results for all variations of simulations and modulations. According to that simulation (14) and (22) are the leaders.
2. Minimum of maximum variance for MMSE estimation error (16) provide best results for communication system with orthogonal precoding matrix selection. The efficiency of such criteria is proven by results at [34]. For the system with subcarriers set selection this criterion occupies the intermediate position;
3. Maximum of mean SNR (8) is ineffective for communication systems with vector channel selection and provide less gain (2-2.5 dB for QPSK and 2.5-4 dB for 16QAM) for orthogonal precoding matrix selection while criteria #7 provided up to 6.5 dB gain;
4. Maximum of capacity (18) is also ineffective and much worse than the results for criteria #4 and #7.

Conclusion

Vector channel selection or precoding matrix selection is always the way to improve the noise immunity of communication systems. The gain for each technique depends on the conditions for the use and chosen search criteria.

In this article we considered a few types of vector channel and precoding matrix selection criteria like common as maximum SNR, maximum channel capacity as well as new criteria. Some of these new ones are empirical such as minimum of maximum MMSE estimation errors and minimum of mean inverse square norm of channel matrix columns.

For efficiency demonstration of both known and proposed criteria an extensive simulation has been done. The simulation results show us that we may provide an efficiency gain over 20 dB for uncorrelated fading channels and 14 dB for real channel with 16 QAM modulation. If there are no option to increase efficiency by vector channel selection, we may use orthogonal precoding matrix to have 2-6 dB for the system.

Proposed empirical criteria of minimum of mean inverse square norm of channel matrix column demonstrates better efficiency than others for communication systems with channel vector selection and for precoding matrices. In comparison with minimum of mean estimation error it has the same efficiency but less computational complexity.

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АНАЛИЗ КРИТЕРИЕВ ДЛЯ ВЫБОРА ВЕКТОРНОГО КАНАЛА ИЛИ МАТРИЦЫ ОРТОГОНАЛЬНОГО ПРЕДВАРИТЕЛЬНОГО КОДИРОВАНИЯ В СИСТЕМАХ СВЯЗИ С МУЛЬТИПЛЕКСИРОВАНИЕМ

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Аннотация

Практически все современные многопользовательские системы связи используют технологию мультиплексирования потоков символов по нескольким параллельным каналам. Число выделяемых каналов пользователю, обычно, меньше общего числа каналов в системе и поэтому всегда есть возможность использовать разные наборы. В этом случае возникает проблема выбора векторного канала для пользователя из общего набора. Не всегда традиционные критерии выбора, используемые для скалярных каналов, такие как, например, максимум среднего отношения сигнал/шум или максимума пропускной способности, обеспечивают наилучший выбор с точки зрения качества передачи информации. Похожая проблема возникает и при выборе различных видов прекодирования в векторных каналах для достижения максимальной энергетической эффективности. Рассмотрено несколько вариантов критериев для выбора векторного канала или ортогонального прекодирования, включая такие как, минимум средней дисперсии ошибки линейного оценивания символа или минимум максимального значения дисперсии. Методом математического моделирования проанализирована эффективность использования разных критериев для системы связи OFDMA при выборе набора поднесущих, а также при выборе матрицы ортогонального прекодирования. Результаты анализа показывают, что эмпирические критерии часто дают лучшие результаты по сравнению с традиционными.

Ключевые слова: *векторный канал, MIMO, критерий выбора, пространственное мультиплексирование, матрица канала*

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