APPLICATION OF RESOLUTION TIME THEORY TO THE DEVELOP AND PERFORMANCE ESTIMATION OF BROADBAND DATA TRANSMISSION SYSTEMS BASED ON BIPOLAR PAM-N SIGNALS UNDER IMPACTION OF CROSSTALK

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The paper presents a new algorithm with linear time complexity, which depends only on the effective channel memory and the number of crosstalk sources, for estimating the resolution time and capacity of a frequency-selective communication channel using a linear receiver and signals with duopolar multi-position pulse amplitude modulation. The main limitation of the presented method is that it can only be used for the case when the durations of channel symbols of the information and interfering signals are the same and the start times of their transmission coincide. From a practical point of view, this method can be used in the analysis of high-speed wired data transmission interfaces in which information is transmitted simultaneously and in parallel over several communication lines that are located sufficiently close to each other. The key features of this method are: 1) a constant number of equations equal to 1, required to be solved to estimate the largest settling time; 2) a linear dependence of the number of equation terms on the effective memory and the number of crosstalk sources; 3) a new, more accurate procedure for estimating the effective channel memory, which allows for simultaneous determination of the resolution time and low boundary capacity estimation; 4) new set of assessments that allows to estimate requirements for symbol synchronization subsystems; 5) new method for estimating the required minimum signal-to-noise ratio.

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Introduction

Nowadays, transformation of society into an information one leads to an exponential growth in the volume of transmitted information [1-4]. While this growth over the last decade has been driven largely by wireless communications systems [5-8], with the advent of data centers in the field of artificial intelligence, there has been a shift in priorities to wired broadband communications systems [9-12]. This is due in particular to the fact that different varieties of such systems are used to ensure the exchange of data between computing devices, sensors and information processing devices [13-15]:

It should be noted that the dilemma faced by developers of this class of systems is that the desire to reduce the bit energy while simultaneously increasing the data transfer rate, on the one hand, reduces the impact of crosstalk, and on the other hand, complicates the signal-noise environment.

Taking into account this fact and that at present the methods of channel equalization are becoming increasingly widespread, for which the issues of the influence of noise on the quality of the restored message at the channel output are quite acute in the construction of such systems, the issue of finding an optimal solution lying between the selection of optimal pre-distortion and optimal signal-noise environment is extremely relevant.

Therefore, it becomes obvious that it is necessary to perform an appropriate analysis that would allow us to determine the optimal set of parameters, which, on the one hand, would ensure optimal values for noise immunity, and, on the other hand, reduce the influence of crosstalk and intersymbol interference to the required level.

Nowadays such analysis is carried out by means of probabilistic model that utilizes cyclostationary signals [10-12], which was produced in the middle of the 20th century [16, 17]. The model itself directly extracts an information signal from a stationary random process and it's widely used to describe noise, by estimating number parameters of signal statistical characteristics that have a repetition period [11]. In a similar way, this approach can be used to analyze crosstalk interference during signal transmission over adjacent transmission lines, caused by the presence of interwire capacitive connections between them [9, 13, 18, 19], since the theory of LTI is also applicable in this case.

However, an analysis of recent papers in this area [9-11, 20] points to a number of significant limitations of existing methods, the main one is the non-polynomial computational complexity when analytical methods are used, impossibility of obtaining the solutions in the case when electromagnetic parameters of the linear path for the information and interfering signals are different; the lack of analytical methods that allow the number of sources of crosstalk to be taken into account is greater than 2.

Based on the above, it becomes obvious that it is relevant to create a new class of methods for estimating the parameters of signal integrity analysis for various implementations of signal reception and processing methods in wired broadband communication systems [21-24]. It is advisable to solve this problem based on the methods of the resolution time theory [14, 15, 25, 27, 28] due to their extremely low computational complexity.

The aim of this study is producing analytical method with extremely low computational complexity for estimating the main parameters of a wired baseband broadband telecommunications system operating with PAM-n-signals under the influence of crosstalk within the framework of the resolution time theory.

The *main limitation* of the developed method is that it can only be used for the case when the durations of channel symbols of the information and interfering signals are the same and the start times of their transmission coincide.

1. Problem Statement

The following peculiarities of real broadband communication systems would be taken into account:

1. Before the moment of time at which the observed process at the output of the frequency-selective channel (FSC) it is non-stationary process, which imposes additional requirements for symbol synchronization subsystem.
2. Primary impact of crosstalk source is taken into account.
3. The adverse effects of crosstalk can be represented as the interfering signal transmission through an LTI system, which parameters characterize the parasitic relationships between transmission lines at the point of signal receiving

1.1 Math model

Taking into account the peculiarities mentioned above, the model of the studied real FSC can be presented as a structural diagram (Fig. 1).

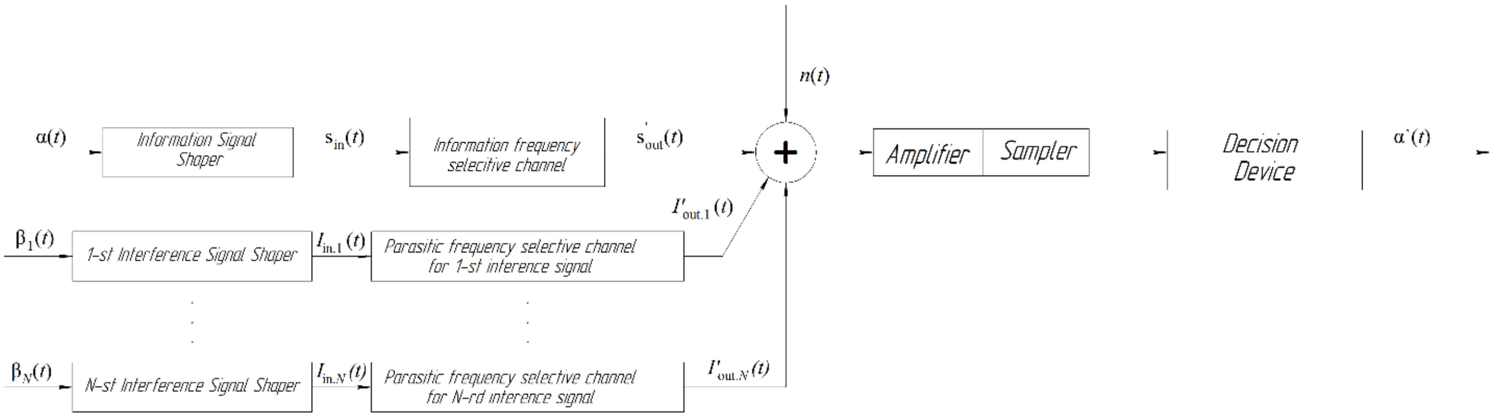

Fig. 1. Block diagram of a considered FSC

The dispersive characteristics of the real channel, through which the information PAM-n_0-signal is transmitted, are determined by the information FSC (InFSC) in the presented block diagram in Fig. 1. The i^{th} crosstalk on PAM-n_0-signal is produced by parasitic interconnections between the communication lines, through one of which the PAM-n_0-signal and through the other the i-th interfering PAM-n_i-signal are transmitted, respectively. Taking into account the results of paper [9], in the receive end the impaction on PAM-n_0-signal the i^{th} crosstalk can be represented as a convolution of the impulse response of LTI that presented the i interfering frequency selective channel (IfFSC) produced by parasitic interconnections, and PAM-n_i-signal.

The shaper in the InFSC generates information PAM-n_0-signal, which can be represented as follows

$$s_{\text{in}}(t)=\sum_{r=1}^{l}M_r g_{\text{sh}_0}\left(t-(r-1)\tau_s\right), \tag{1}$$

under the following sequence impaction on its input

$$\alpha(t)=\sum_{r=1}^{l}M_r\delta\left(t-(r-1)\tau_s\right). \tag{2}$$

At the same time shaper in the i- IfFSC generates PAM-n_i-signal, which can be represented as follows

$$I_{\text{in.i}}(t)=\sum_{r=1}^{l}A_{i,r}g_{\text{sh}_i}\left(t-(r-1)\tau_s\right), \tag{3}$$

under the following sequence impaction on its input

$$\beta_i(t)=\sum_{r=1}^{l}A_{i,r}\delta\left(t-(r-1)\tau_s\right). \tag{4}$$

Here $\delta(t)$ is Dirac delta function; $g_{\text{sh}_0}(t)$ and $g_{\text{sh}_i}(t)$ are partial pulses at the output of shapers in the InFSC and i-IfFSC produced by $\delta(t)$ acting on their inputs, respectively; τ_s is a symbol time duration for PAM-n_0- and PAM-n_i- signal; $M_r\in\mathbf{M}$ and $A_{i,r}\in\mathbf{A}_i$ are amplitudes of r^{th} pulse of PAM-n_0- signal and PAM-n_i- signal, respectively; $i\in\overline{1,N}$; N is sources of interfering PAM-signals; l is number of channel symbols in transmitted sequence.

The values of signal constellation (SC) of PAM-n_0- signal $\mathbf{M}$ is defined in the following way:

$$\mathbf{M}=\left\{M_{\text{sc}_k}\right\}_{k=1}^{n_0}=\left\{M_{\text{sc}_k}=\left(k_0-\|n_0/2\|-0.5\left(1-n_0 \bmod 2\right)\right)\Delta M_{\text{st}},k_0=\overline{1,n_0}\right\}, \tag{5}$$

and for PAM-n_i- signal as follows

$$\mathbf{A}_i=\left\{A_{\text{sc}_{k_i},i}\right\}_{k_i=1}^{n_i}=\left\{A_{\text{sc}_{k,i}}=\left(k_i-\|n_i/2\|-0.5\left(1-n_i \bmod 2\right)\right)\Delta A_{\text{st}_i},k_i=\overline{1,n_i}\right\} \tag{6}$$

Here ΔM_{st} and ΔA_{st_i} are step between the closest values of the SC elements of PAM-n_0- and PAM-n_i- signals, respectively; $n_\bullet \bmod 2$ is remainder from division $n_\bullet$ by 2.

It is obvious that the following conditions are also met true for the greatest absolute values of considered SC $M_{\max}$ and $A_{\max_i}$:

$$\left|M_{\text{sc}_1}\right|=\left|M_{\text{sc}_{n_0}}\right|=M_{\max};\quad \left|A_{\text{sc}_{1,i}}\right|=\left|A_{\text{sc}_{n_i,i}}\right|=A_{\max_i}, \tag{7}$$

To achieve the capacity lower bound Gray code is used and assertion is made that each value of the channel symbol in the sequences does not depend on the values of the preceding symbols and takes on any of the values of the signal constellation used with equal probability according to the papers [14, 15, 25].

The process at the input of amplifier on the receive end can be presented in the following form using the results obtained in papers [9, 19, 13]:

$$\begin{aligned}s_\Sigma(t)&=k_{\text{los}_0}\left(s_{\text{in}}(t)*h_0(t)\right)+\sum_{i=1}^{N}k_{\text{los}_i}\left(I_{\text{in.i}}(t)*h_i(t)\right)+n(t)=\\&=k_{\text{los}_0}s'_{\text{out}}(t)+\sum_{i=1}^{N}k_{\text{los}_i}I'_{\text{out.i}}(t)+n(t)=\\&=k_{\text{los}_0}\sum_{r=1}^{l}M_rP\left(t-(r-1)\tau_s\right)+\sum_{i=1}^{N}k_{\text{los}_i}\sum_{r_i=1}^{l}A_{i,r_i}I_i\left(t-(r-1)\tau_s\right)+n(t)\end{aligned} \tag{8}$$

Here $*$ is convolution operation; $P(t)$ and $I_i(t)$ are the responses of InFSC and i-IfFSC on partial pulse $g_{\text{sh}_0}(t)$ and $g_{\text{sh}_i}(t)$, respectively; $n(t)$ is additive white noise (AWN); k_{los_0} is losses introduced by the propagation medium that defines InFSC; k_{los_i} is losses introduced by the propagation medium that defines i-IfFSC; $h_0(t)$ and $h_i(t)$ are impulse responses of InFSC and i-IfFSC, respectively.

Parameters of additive white noise [26] are defined as follows:

– the power spectrum

$$N(f)=\begin{cases}N_0/2, & |f|\le f_{\max};\\ 0, & |f|>f_{\max};\end{cases} \tag{9}$$

– he correlation function

$$R_N(\tau)=N_0\frac{\sin\left(2\pi f_{\max}\tau\right)}{2\pi\tau}, \tag{10}$$

where $f_{\max}=1/\tau_s$.

Each j^{th} cross-section of $n(t)$ is a random variable n_j with PDF [26] is

$$f_{n_j}\left(n_j\right)=\left(\sigma\sqrt{2\pi}\right)^{-1}\exp\left(-0.5\left(n_j/\sigma\right)^2\right), \tag{11}$$

where $\sigma=\sqrt{N_0 f_{\max}}$ is standard deviation of a random variable n_j.

The gain of adaptive amplifier $k_A=k_{A_1}k_{A_2}$ is adjusted before the transmission start so that the following relationships are met true [25]

$$k_{A_1} = k_{\mathrm{los}_0}{}^{-1} \tag{12}$$

$$k_{A_2} = \begin{cases} \dfrac{\max g_{\mathrm{sh}_0}(t)}{\max P(t)} \text{ if } \max g_{\mathrm{sh}_0}(t) \ge \max P(t); \\ 1 \text{ if } \max g_{\mathrm{sh}_0}(t) < \max P(t). \end{cases} \tag{13}$$

According the last expressions, the observed process at the amplifier output has the following form

$$s_{\mathrm{amp}}(t) = k_{\mathrm{A}} s_{\Sigma}(t) = k_{\mathrm{A}_2} s'_{\mathrm{out}}(t) + k_{\mathrm{A}} \sum_{i=1}^{N} k_{\mathrm{los}_i} I'_{\mathrm{out.i}}(t) + k_{\mathrm{A}} n(t) = \\ = \sum_{r=1}^{l} \left[M_r P'(t-(r-1)\tau_s) + \sum_{i=1}^{N} A_{i,r} I'_i(t-(r-1)\tau_s) \right] + n'(t), \tag{14}$$

where $P'(t-(r-1)\tau_{\mathrm{s}}) = k_{\mathrm{A}_2} P(t-(r-1)\tau_{\mathrm{s}})$;

$I'_i(t-(r-1)\tau_{\mathrm{s}}) = k_{\mathrm{A}} k_{\mathrm{los}_i} I_i(t-(r-1)\tau_{\mathrm{s}})$; $n'(t) = k_{\mathrm{A}} n(t)$.

The signal on the input of decision device can be represented as follows:

$$s_{\mathrm{sp}}(t) = \sum_{j=1}^{l} \delta(t - j\tau_{\mathrm{s}}) s_{\mathrm{amp}}((j-1)\tau_{\mathrm{s}} + \Delta T_{\mathrm{s}}) \tag{15}$$

Here $\Delta T_{\mathrm{s}} \in (0; \tau_{\mathrm{s}})$ is the shift in the retrieving moment of the transmitted symbol in the sequence, caused by the instability of the sampler and can also be represented as follows

$$\Delta T_{\mathrm{s}} = \begin{cases} \Delta T_{\mathrm{opt}_d} + \Delta T'_{\mathrm{s}} \text{ if } d < G+1 \\ \Delta T_{\mathrm{opt}} + \Delta T'_{\mathrm{s}} \text{ if } d \ge G+1 \end{cases} \tag{16}$$

where $\Delta T_{\mathrm{opt}_d}$ and ΔT_{opt} are the optimal time shift for retrieving information about d^{th} channel symbol for the following conditions $d < G+1$ and $d \ge G+1$, respectively; $\Delta T'_{\mathrm{s}}$ is a centered random variable with respect to $\Delta T_{\mathrm{opt}_d}$ и ΔT_{opt}.

The recovering of the channel symbol values in the sequence for PAM-n_0-signal on the receiving end is performed in accordance with the following rule

$$M_{\mathrm{rec}_d} = M_p \big|_{p=p'}, \tag{17}$$

where $p' \in \overline{1,n} : f(p',d) = \min_{p \in \overline{1,n}} \left| s_{\mathrm{sp}}(d\tau_{\mathrm{s}}) - M_p \right|$; $d = \overline{1,l}$ is number of the recovering channel symbol.

The reconstructed information sequence at the receive end has the form

$$\alpha'(t) = \sum_{r=1}^{l} M_{\mathrm{rec}_r} \delta(t - r\tau_{\mathrm{s}}) \tag{18}$$

1.2 Estimations

In this paper following estimations are used to assessment the signal integrity:

1. The greatest settling time $t_{\mathrm{set}_d} = \left\{ \tau_{\mathrm{w.st}_{d,k}} \right\}_{k=1}^{S_{\mathrm{w}}} \cup \left\{ \tau_{\mathrm{w.end}_{d,k}} \right\}_{k=1}^{S_{\mathrm{w}}}$ for the d^{th} channel symbol in the information sequence for given permissible settling error Δ_{pm} [25]. Here $\tau_{\mathrm{w.st}_{d,k}}$ and $\tau_{\mathrm{w.end}_{d,k}}$ are the symbol durations at which the k-th transparency window starts and ends for the d^{th} symbol, respectively; S_{w} is a number of "transparency" windows for the d-th symbol.

2. The resolution time $t_{\mathrm{res}} = \left\{ \tau_{\mathrm{w.st}_k} \right\}_{k=1}^{W} \cup \left\{ \tau_{\mathrm{w.end}_k} \right\}_{k=1}^{W}$. Here $\tau_{\mathrm{w.st}_k}$ and $\tau_{\mathrm{w.end}_k}$ are the channel symbol durations at which the k^{th} "transparency" window starts and ends for any d-th channel symbol in the transmitted information sequence when condition $d \ge G+1$ is met true; G is effective memory of the considered FSC [25]; W is a number of "transparency" windows;

The resolution time t_{res} estimation is based on the validity of the transposition principle for considered FSC, due to which the statement $|T| \dot{\le} \varepsilon_{\mathrm{res}}$ is met true based on the following rule

$$|T| \dot{\le} \varepsilon_{\mathrm{res}} = \begin{cases} \text{true if } (T \ne \varnothing) \wedge (\max|T| \le \varepsilon_{\mathrm{res}}); \\ \text{false if } (T = \varnothing) \vee (\max|T| > \varepsilon_{\mathrm{res}}); \end{cases} \tag{19}$$

where $|T| = \{ |t| \, | \, t \in T \}$. Here T is a set, which is defined in the following way

$$T = t_{\mathrm{res}} \dot{-} t_{\mathrm{set}_{G+1}} = \begin{cases} \left\{ \Delta\tau_{\mathrm{w.st}_k} \right\}_{k=1}^{W} \bigcup \left\{ \Delta\tau_{\mathrm{w.end}_k} \right\}_{k=1}^{W} \text{ if } \|t_{\mathrm{res}}\| = \|t_{\mathrm{set}_{G+1}}\|; \\ \varnothing \text{ if } \|t_{\mathrm{res}}\| \ne \|t_{\mathrm{set}_{G+1}}\|. \end{cases} \tag{20}$$

Here $\| \, . \, \|$ is a set cardinality; $\Delta\tau_{\mathrm{w.st}_k} = \tau_{\mathrm{w.st}_k} - \tau_{\mathrm{w.st}_{G+1,k}}$ $\Delta\tau_{\mathrm{w.end}_k} = \tau_{\mathrm{w.end}_k} - \tau_{\mathrm{w.end}_{G+1,k}}$; $t_{\mathrm{set}_{G+1}} = t_{\mathrm{set}_d} \big|_{d=G+1}$.

The set of symbol durations T_{res} that determines the resolution time t_{res} with precision $\varepsilon_{\mathrm{res}}$ can be defined as follows:

$$T_{\mathrm{res}} = \left\{ \tau_{\mathrm{s}} \in \mathbb{R} \middle| (S_{\mathrm{A}}(\tau_{\mathrm{s}}) + \Delta \le Q_{\mathrm{A}}) \wedge (Q_{\mathrm{A}} < 0.5\Delta M_{\mathrm{st}}) \wedge (Q_{\mathrm{A}} \to 0.5\Delta M_{\mathrm{st}}) \right\} = \\ \bigcup_{k=1}^{S_W} \left[\tau_{\mathrm{w.st}_{G+1,k}} ; \tau_{\mathrm{w.end}_{G+1,k}} \right] \tag{21}$$

where

$$S_{\mathrm{A}}(\tau_{\mathrm{s}}) = \max \left| \Delta_{\mathrm{set}}((G+1)\tau_{\mathrm{s}}) \right| = \max \times \\ \times \left| k_{\mathrm{A}_2} s'_{\mathrm{out}}((G+1)\tau_{\mathrm{s}}) + k_{\mathrm{A}} \sum_{i=1}^{N} k_{\mathrm{los}_i} I'_{\mathrm{out.i}}((G+1)\tau_{\mathrm{s}}) - M_{G+1} \right|. \tag{22}$$

Here Q_{A} is a parameter that characterizes the degree of non-ideality quantization of the decision device; $\Delta = k_A \sigma F^{-1}(1 - BER \times \log_2 n_0)$ is an absolute value of the anomalous measurement error [25]; $F^{-1}(.)$ is inverse function for the CDF of the standard normal PD law [25]; $\Delta_{\mathrm{set}}(\, . \,)$ is settling error.

It should be noted that $S_{\mathrm{A}}(t_{\mathrm{set}_{G+1}}) = \Delta_{\mathrm{pm}}$, where Δ_{pm} is permissible settling error [25].

3. Optimal time shift ΔT_{opt} for retrieving information about d^{th} channel symbol within the following condition $d \geq G+1$ is true at which minimum settling error is achieved

$$\Delta T_{\text{opt}} = \underset{t_{\text{sh}_{G+1}}\left(\tau_{\text{w.st}_{G+1,1}}\right)}{\arg\min} \times$$
$$\times\left\{S'_{\text{A}}\left(\tau_{\text{w.st}_{G+1,1}}, t_{\text{sh}_{G+1}}\left(\tau_{\text{w.st}_{G+1,1}}\right), G+1\right) \middle| S'_{\text{A}}\left(\tau_{\text{w.st}_{G+1,1}}, t_{\text{sh}_{G+1}}\left(\tau_{\text{w.st}_{G+1,1}}\right), G+1\right) < Q_{\text{A}}\right\} \quad (23)$$

and optimal time shift ΔT_{opt_d} within the following condition $d < G+1$ is true

$$\Delta T_{\text{opt}_d} = \underset{t_{\text{sh}_d}\left(\tau_{\text{w.st}_{G+1,1}}\right)}{\arg\min} = \left\{S'_{\text{A}}\left(\tau_{\text{w.st}_{G+1,1}}, t_{\text{sh}_d}\left(\tau_{\text{w.st}_{G+1,1}}\right), d\right) \middle| \times\right.$$
$$\left.\times S'_{\text{A}}\left(\tau_{\text{w.st}_{G+1,1}}, t_{\text{sh}_d}\left(\tau_{\text{w.st}_{G+1,1}}\right), d\right) < Q_{\text{A}}\right\} \quad (24)$$

Here $t_{\text{sh}_d}\left(\tau_{\text{w.st}_{G+1,1}}\right)$ and $t_{\text{sh}_{G+1}}\left(\tau_{\text{w.st}_{G+1,1}}\right)$ are time shifts relative to the transmission start of the received d^{th} and $(G+1)$-th channel symbols in sequence at symbol duration $\tau_{\text{w.st}_{G+1,1}}$, respectively

$$S'_{\text{A}}(\kappa, \lambda, d) = \max\left|\Delta_{\text{set}}(d\kappa + \lambda)\right| =$$
$$= \max\left|k_{\text{A}_2} s'_{\text{out}}(d\kappa + \lambda) + k_{\text{A}} \sum_{i=1}^{N} k_{\text{los}_i} I'_{\text{out}.i}(d\kappa + \lambda) - M_d\right| \quad (25)$$

5. The permissible time shift $T_{\text{sh.pm}}$ is a time shift relative to the transmission start of the received channel symbol, which is defined in the following way:

- in case of analysis of considered process at the input of adaptive amplifier is made when it becomes cyclostationary ($d \geq G+1$)

$$T_{\text{sh.pm}} = t_{\text{sh.pm}_{G+1}}\left(\tau_{\text{w.st}_{G+1,1}}\right), \quad (26)$$

- in the case analysis of considered process at the input of adaptive amplifier is made when it is a mixture of nonstationary and cyclostationary process

$$T_{\text{sh.pm}} = \min_d\left(\left\{t_{\text{sh.pm}_d}\left(\tau_{\text{w.st}_{G+1,1}}\right) \middle| d = \overline{1, G+1}\right\}\right), \quad (27)$$

where $t_{\text{sh.pm}_d}\left(\tau_{\text{w.st}_{G+1,1}}\right) = \min\left\{T_{\text{sh}}\left(\tau_{\text{w.st}_{G+1,1}}\right)\right\}$ and T_{sh} is obtained by solving the following equation

$$S'_{\text{A}}\left(\tau_{\text{w.st}_{G+1,1}}, T_{\text{sh}}\left(\tau_{\text{w.st}_{G+1,1}}\right), d\right) = \Delta_{\text{pm}}$$

6. The largest spread of optimal time shifts for retrieving information

$$\delta_{\text{spr}} = \max_{d \in 1, G+1} \frac{\left|\Delta T_{\text{opt}_d} - \Delta T_{\text{opt}}\right|}{\Delta T_{\text{opt}}} \quad (28)$$

The minimum required signal-to-noise ratio (SNR) for given BER value at which lower bound capacity estimation is achieved for given BER value when the optimal time shift for retrieving information ΔT_{opt} is used and $\tau_{\text{s}} = \tau_{\text{w.st}_1}$

$$\text{Min } SNR = \max_d q \quad (29)$$

Here $q = \left\{\text{Min } SNR_d \middle| d = \overline{1, G+1}\right\}$; $\text{Min } SNR_d$ is determined based on the use of the results of paper [25] as follows

$$\text{Min } SNR_d = 10\log_{10}\frac{P_{\text{mm}}\left(d\tau_{\text{w.st}_1} + \Delta T_{\text{opt}}\right)}{(k_A\sigma)^2} =$$
$$= 10\log_{10}\frac{\frac{1}{n_0}\sum_{k=1}^{n_0}\left(\left|M_{\text{sc}_k}\right| - \Delta'_{\max}\left(0, d, \Delta T_{\text{opt}}\right)\right)^2}{\left(\frac{\Delta}{F^{-1}\left(1 - BER \times \log_2 n_0\right)}\right)^2} \quad (30)$$

where

$$P_{\text{mm}}\left(d\tau_{\text{w.st}_1} + \Delta T_{\text{opt}}\right) = \frac{1}{n_0}\sum_{k=1}^{n_0}\left(\left|M_{\text{sc}_k}\right| - \Delta'_{\max}\left(0, d, \Delta T_{\text{opt}}\right)\right)^2$$

is mean minimum power of signal at the d^{th} channel symbol value retrieving moment; the maximum value of settling error at optimal time shift ΔT_{opt} for retrieving information about d^{th} channel symbol

$$\Delta'_{\max}\left(0, d, \Delta T_{\text{opt}}\right) = \max \times$$
$$\times\left|k_{\text{A}_2} s'_{\text{out}}\left(d\tau_{\text{w.st}_1} + \Delta T_{\text{opt}}\right) + k_{\text{A}} \sum_{i=1}^{N} k_{\text{los}_i} I'_{\text{out}.i}\left(d\tau_{\text{w.st}_1} + \Delta T_{\text{opt}}\right) - M_d\right|$$

Taking into account that $\Delta + \Delta'_{\max}\left(0, d, \Delta T_{\text{opt}}\right) = Q_{\text{A}}$ the final expression for $\text{Min } SNR_d$ will take the form

$$\text{Min } SNR_d = 20\log_{10}\frac{F^{-1}\left(1 - BER \times \log_2 n_0\right)}{Q_{\text{A}} - \Delta'_{\max}\left(0, d, \Delta T_{\text{opt}}\right)}\frac{1}{\sqrt{n_0}} \times$$
$$\times\sqrt{\sum_{k=1}^{n_0}\left(\left|M_{\text{sc}_k}\right| - \Delta'_{\max}\left(0, d, \Delta T_{\text{opt}}\right)\right)^2} \quad (31)$$

7. The minimum SNR $\text{Min } SNR(p)$ in the presence of the maximum feducial symbol sampler instability $p = \Delta t_{\text{inst}} / \tau_{\text{w.st}_1}$ at the symbol duration $\tau_{\text{s}} = \tau_{\text{w.st}_1}$ and optimal time shift ΔT_{opt} for retrieving information about channel symbol, here Δt_{inst} is temporal instability of the sampler. Taking into account the results of paper [25] $\text{Min } SNR(p)$ is defined as follows

$$\text{Min } SNR(p) = \max_d q', \quad (32)$$

here $q' = \left\{\text{Min SNR}(p, d) \middle| d = \overline{1, G+1}\right\}$; $\text{Min SNR}(p, d)$ is determined based on the use of the results of paper [22] and taking into account that $\Delta + \Delta'_{\max}\left(\gamma, d, \Delta T_{\text{opt}}\right) = Q_{\text{A}}$ as follows

$$\mathrm{Min\ SNR}(p,d)=$$
$$=\begin{cases}\varnothing \text{ if } \left(\Delta T_{\text{opt}}-p\tau_{\text{w.st}_1}<0\right)\wedge\left(\max\limits_{\pm p}\Delta'_{\max}\left(\pm p,d,\Delta T_{\text{opt}}\right)\geq Q_{\text{A}}\right)\wedge\left(\Delta T_{\text{opt}}+p\tau_{\text{w.st}_1}>\tau_{\text{w.st}_1}\right);\\ 20\log_{10}\dfrac{F^{-1}\left(1-BER\times\log_2 n_0\right)}{\left(Q_{\text{A}}-\max\limits_{\pm p}\Delta'_{\max}\left(\pm p,d,\Delta T_{\text{opt}}\right)\right)\sqrt{n_0}}\sqrt{\sum\limits_{k=1}^{n_0}\left(\left|M_{\text{sc}_k}\right|-\max\limits_{\pm p}\Delta'_{\max}\left(\pm p,d,\Delta T_{\text{opt}}\right)\right)^2}\\ \text{if } \left(\Delta T_{\text{opt}}-p\tau_{\text{w.st}_1}\geq 0\right)\wedge\left(\max\limits_{\pm p}\Delta'_{\max}\left(\pm p,d,\Delta T_{\text{opt}}\right)<Q_{\text{A}}\right)\wedge\left(\Delta T_{\text{opt}}+p\tau_{\text{w.st}_1}\leq\tau_{\text{w.st}_1}\right),\end{cases}\quad(33)$$

where

$$\Delta'_{\max}\left(\pm p,d,\Delta T_{\text{opt}}\right)=\max\Big|k_{A_2}s'_{\text{out}}\left(d\tau_{\text{w.st}_1}+\Delta T_{\text{opt}}\pm p\tau_{\text{w.st}_1}\right)+$$
$$+k_{\text{A}}\sum_{i=1}^{N}k_{\text{los}_i}I'_{\text{out.i}}\left(d\tau_{\text{w.st}_1}+\Delta T_{\text{opt}}\pm p\tau_{\text{w.st}_1}\right)-M_d\Big|.$$

8. The attenuation SA for information signal at the output of the considered FSC, caused by the incomplete amplitude settling of the partial signal pulse due its short duration and transient processes. SA is determined by the following relationship

$$SA=-20\log_{10}\left(k_{A_2}\right).\quad(34)$$

9. Lower bound capacity estimation

$$C_{\text{lb}}\left(n_0\right)=\frac{1}{\tau_{\text{w.st}_1}}\log_2 n_0\quad(35)$$

where $\tau_{\text{w.st}_1}\subseteq t_{\text{res}}$ is a symbol time duration at which the 1st "transparency" window starts.

10. Auxiliary lower bound capacity estimation that taking into account nonstationary nature of considered process (14) until the it becomes cyclostationary process in the case when the data rate is significantly exceeds the Faster Than Nyquist rate

$$C_{\text{alb}}\left(n_0\right)=\frac{1}{\max\left\{\left\{\tau_{\text{w.st}_1}\right\}\bigcup\left\{\tau_{\text{w.st}_{d,1}}\right\}_{d=1}^{G}\right\}}\log_2 n_0\quad(36)$$

1.3. The List of Problems to be Solved

Analyzing the mathematical model of a composite FSC presented in the previous section of this paper and taking into account the goal of this paper, the following problems that need to be solved to achieve it should be formulated:

1) An equation that allows calculating the greatest settling time for the information parameter of PAM-n_0-signal at the output of the adaptive amplifier for a transmitted information sequence with an arbitrary number of symbols in the presence of crosstalk should be obtained.

2) An expression that allows calculating the maximum settling error at the output of the adaptive amplifier for arbitrary time shift for considered symbol duration for given number of symbols in information sequence in the presence of crosstalk should be obtained. An equation that allows calculating the permissible time shift for given value of symbol duration for the information parameter of PAM-n_0-signal at the output of the adaptive amplifier for a transmitted information sequence with an arbitrary number of symbols in the presence of crosstalk should be obtained.

3) The method for effective memory estimation in the presence of crosstalk should be obtained for considered channel math model.

4) Develop an algorithm to estimating the main parameters of the system presented in Section 1.2 of this paper, operating in the frequency-selective communication channel under consideration.

2. The Problem Solutions

2.1 The solution of 1st problem

Let's obtain an expression that allows to estimate the amplitude largest settling time for the d-th channel symbol of the PAM-n_0-signal at the output of the considered FSC with a symbol duration $\tau_{\text{s}}=t_{\text{set}_d}$.

First of all, we obtain an expression that allows to estimate the settling error $\Delta_{\text{set}}\left(d\tau_{\text{s}}\right)$ for the d-th channel symbol without taking into account the influence of the AWN and influence of sampler instability $\Delta T_{\text{s}}=0$ and channel symbol duration τ_{s}. To do this, lets transform the expression (14) using the following substitutions: $l=d$, $t=l\tau_{\text{s}}=d\tau_{\text{s}}$, $s_{\text{amp}}\left(d\tau_{\text{s}}\right)-n'\left(d\tau_{\text{s}}\right)=M_d+\Delta_{\text{set}}\left(d\tau_{\text{s}}\right)$.

As a result, we get

$$M_d+\Delta_{\text{set}}\left(d\tau_{\text{s}}\right)=$$
$$=\sum_{r=1}^{d}\left[M_rP'\left(d\tau_{\text{s}}-(r-1)\tau_{\text{s}}\right)+\sum_{i=1}^{N}A_{i,r}I'_i\left(d\tau_{\text{s}}-(r-1)\tau_{\text{s}}\right)\right]=$$
$$=\sum_{r=1}^{d}\left[M_rP_{r,d}\left(\tau_{\text{s}}\right)+\sum_{i=1}^{N}A_{i,r}I_{i,r,d}\left(\tau_{\text{s}}\right)\right],\quad(37)$$

where $P_{r,d}\left(\tau_{\text{s}}\right)=P'\left((d-r+1)\tau_{\text{s}}\right)$, $I_{i,r,d}\left(\tau_{\text{s}}\right)=I'_i\left((d-r+1)\tau_{\text{s}}\right)$.

From equality (37) follows expression for the d-th channel symbol settling error $\Delta_{\text{set}}\left(d\tau_{\text{s}}\right)$

$$\Delta_{\text{set}}\left(d\tau_{\text{s}}\right)=\sum_{r=1}^{d}\left[M_rP_{r,d}\left(\tau_{\text{s}}\right)+\sum_{i=1}^{N}A_{i,r}I_{i,r,d}\left(\tau_{\text{s}}\right)\right]-M_d=$$
$$=\sum_{r=1}^{d-1}\left[M_rP_{r,d}\left(\tau_{\text{s}}\right)+\sum_{i=1}^{N}A_{i,r}I_{i,r,d}\left(\tau_{\text{s}}\right)\right]+\quad(38)$$
$$+M_d\left[P_{d,d}\left(\tau_{\text{s}}\right)-1\right]+\sum_{i=1}^{N}A_{i,d}I_{i,d,d}\left(\tau_{\text{s}}\right)$$

The next step of 1st problem solution is the following equation to be solved

$$\max\left|\Delta_{\text{set}}\left(dt_{\text{set}_d}\right)\right|=\Delta_{\text{pm}},\quad(39)$$

where the maximization is made over all combination of channel symbols values in transmitted information sequence.

Obviously, to solve this equation it is necessary to analyze the expression $\left|\Delta_{\text{set}}\left(dt_{\text{set}_d}\right)\right|$ for extrema, taking into account the following condition $t_{\text{set}_d}<\infty$. To do this, we solve the following system using expression (38) and substitution $\tau_{\text{s}}=t_{\text{set}_d}$

$$\left.\begin{aligned}&\partial\left|\Delta_{\mathrm{set}}\left(dt_{\mathrm{set}_d}\right)\right|/\partial M_1=0\\&\quad\vdots\\&\partial\left|\Delta_{\mathrm{set}}\left(dt_{\mathrm{set}_d}\right)\right|/\partial M_d=0\\&\partial\left|\Delta_{\mathrm{set}}\left(dt_{\mathrm{set}_d}\right)\right|/\partial A_{1,1}=0\\&\quad\vdots\\&\partial\left|\Delta_{\mathrm{set}}\left(dt_{\mathrm{set}_d}\right)\right|/\partial A_{N,d}=0\end{aligned}\right\}\Leftrightarrow\left.\begin{aligned}&P_{1,d}\left(t_{\mathrm{set}_d}\right)=0\\&\quad\vdots\\&P_{d-1,d}\left(t_{\mathrm{set}_d}\right)=0\\&P_{d,d}\left(t_{\mathrm{set}_d}\right)=1\\&I_{1,1,d}\left(t_{\mathrm{set}_d}\right)=0\\&\quad\vdots\\&I_{N,d,d}\left(t_{\mathrm{set}_d}\right)=0\end{aligned}\right\}\qquad(40)$$

Analyzing system (40) we can assert that the extrema exists only for boundary values of SCs due to the fact that considered system has no solutions.

Using the obtained results and approach presented in [14,15] let's modify the equality (39) To do this, first we represent the expression $\left|\Delta_{\mathrm{set}}\left(d\tau_{\mathrm{s}}\right)\right|$ as follows:

$$\left|\Delta_{\mathrm{set}}\left(d\tau_{\mathrm{s}}\right)\right|=$$

$$=\Bigg|\underbrace{\sum_{z_1=1}^{d-1}F_+\left(M_{z_1}P_{z_1,d}\left(\tau_{\mathrm{s}}\right)\right)+F_+\left(M_{d'}\left(P_{d,d'}\left(\tau_{\mathrm{s}}\right)-1\right)\right)+\sum_{i_1=1}^{N}\sum_{r=1}^{d}F_+\left(A_{i_1,r}I_{i_1,r,d}\left(\tau_{\mathrm{s}}\right)\right)}_{S_+\left(\tau_{\mathrm{s}}\right)\ge 0}+$$

$$+\underbrace{\sum_{z_2=1}^{d-1}F_-\left(M_{z_2}P_{z_2,d}\left(\tau_{\mathrm{s}}\right)\right)+F_-\left(M_{d''}\left(P_{d,d''}\left(\tau_{\mathrm{s}}\right)-1\right)\right)+\sum_{i_2=1}^{N}\sum_{r=1}^{d}F_-\left(A_{i_2,r}I_{i_2,r,d}\left(\tau_{\mathrm{s}}\right)\right)}_{S_-\left(\tau_{\mathrm{s}}\right)\le 0}\Bigg|=$$

$$=\left|S_+\left(\tau_{\mathrm{s}}\right)+S_-\left(\tau_{\mathrm{s}}\right)\right|\qquad(41)$$

Here $F_+\left(x\right)=\left|x\right|\mathrm{sgn}\left(\mathrm{sgn}\left(x\right)+1\right)$; $F_-\left(x\right)=\left|x\right|\mathrm{sgn}\left(\mathrm{sgn}\left(x\right)-1\right)$; $\mathrm{sgn}\left(x\right)$ is a signum function; $d'=d''=d$.

Analysis of equality (41) taking into account $M_{z_1}=\mathrm{var}$; $M_{z_2}=\mathrm{var}$; $M_{d'}=\mathrm{var}$; $M_{d''}=\mathrm{var}$; $A_{i_1,r}=\mathrm{var}$; $A_{i_2,r}=\mathrm{var}$ allows to assert that $\forall\tau_{\mathrm{s}},\forall d:\left|\Delta_{\mathrm{set}}\left(d\tau_{\mathrm{s}}\right)\right|\to\max$ is met true if one of the following conditions should be correct:

$$\left(S_+\left(\tau_{\mathrm{s}}\right)\to\max\right)\wedge\left(S_-\left(\tau_{\mathrm{s}}\right)=0\right);\qquad(42)$$

$$\left(S_+\left(\tau_{\mathrm{s}}\right)=0\right)\wedge\left(\left|S_-\left(\tau_{\mathrm{s}}\right)\right|\to\max\right).\qquad(43)$$

The values of term elements of the polynomial in expression (35) at which the above conditions are achieved are presented in Table 1.

Table 1

Values of term elements in expression (41) at which conditions (42) and (43) are meet true

Considered term elements of the polynomial in (41)	The element values at which conditions are meet true			
	condition (42)		condition (43)	
	variant 1	variant 2	variant 1	variant 2
$A_{i_1,r}$	$A_{\mathrm{sc}_{1,i}}$	$A_{\mathrm{sc}_{n_i,i}}$	$A_{\mathrm{sc}_{1,i}}$	$A_{\mathrm{sc}_{n_i,i}}$
$A_{i_2,r}$				
M_{d_1}	M_{sc_1}	$M_{\mathrm{sc}_{n_0}}$	M_{sc_1}	$M_{\mathrm{sc}_{n_0}}$
M_{d_2}				
M_{z_1}				
M_{z_2}				
$P_{z_1,d}\left(\tau_{\mathrm{s}}\right)$	<0	≥ 0	≥ 0	<0
$P_{z_2,d}\left(\tau_{\mathrm{s}}\right)$				
$P_{d,d'}\left(\tau_{\mathrm{s}}\right)-1$				
$P_{d,d''}\left(\tau_{\mathrm{s}}\right)-1$				
$I_{i_1,r,d}\left(\tau_{\mathrm{s}}\right)$				
$I_{i_2,r,d}\left(\tau_{\mathrm{s}}\right)$				

Taking into account (7) the expression for $\Delta_{\max}\left(d\tau_{\mathrm{s}}\right)$ takes the following form

$$\Delta_{\max}\left(d\tau_{\mathrm{s}}\right)=\max\left|\Delta_{\mathrm{set}}\left(d\tau_{\mathrm{s}}\right)\right|=M_{\max}\left(\sum_{r=1}^{d-1}\left|P_{r,d}\left(\tau_{\mathrm{s}}\right)\right|+\left|\left(P_{d,d}\left(\tau_{\mathrm{s}}\right)-1\right)\right|\right)+$$
$$+\sum_{i=1}^{N}A_{\max_i}\sum_{r=1}^{d}\left|I_{i,r,d}\left(\tau_{\mathrm{s}}\right)\right|\qquad(44)$$

Taking into account expression (44) the equality $\max\left|\Delta_{\mathrm{set}}\left(dt_{\mathrm{set}_d}\right)\right|=\Delta_{\mathrm{pm}}$ will take the form

$$\Delta_{\mathrm{pm}}=M_{\max}\left(\sum_{r=1}^{d-1}\left|P_{r,d}\left(t_{\mathrm{set}_d}\right)\right|+\left|\left(P_{d,d}\left(t_{\mathrm{set}_d}\right)-1\right)\right|\right)+$$
$$+\sum_{i=1}^{N}A_{\max_i}\sum_{r=1}^{d}\left|I_{i,r,d}\left(t_{\mathrm{set}_d}\right)\right|.\qquad(45)$$

2.2 The solution of 2nd problem

To solve the 2nd problem, the expression for settling error $\Delta_{\mathrm{set}}\left(d\tau'_{\mathrm{s}_d}+t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)$ for the *d*-th channel symbol with symbol duration $\tau'_{\mathrm{s}_d}\in\bigcup_{k=1}^{\mathrm{S_w}}\left[\tau_{\mathrm{w.st}_{d,k}};\tau_{\mathrm{w.end}_{d,k}}\right]$ should be obtained.

To do this, we transform the equality (14) using the following substitutions $l=d+1$, $t=d\tau'_{\mathrm{s}_d}+t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)$, $s_{\mathrm{amp}}\left(d\tau'_{\mathrm{s}_d}+t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)-n'\left(d\tau'_{\mathrm{s}_d}+t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)=M_d+\Delta_{\mathrm{set}}\left(d\tau'_{\mathrm{s}_d}+t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)$. After simplifications we obtain the following expression:

$$\Delta_{\mathrm{set}}\left(d\tau'_{\mathrm{s}_d}+t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)=\sum_{r=1}^{d-1}M_rP'_{r,d+1}\left(\tau'_{\mathrm{s}_d},t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)+$$
$$+M_d\left[P'_{d,d+1}\left(\tau'_{\mathrm{s}_d},t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)-1\right]+M_{d+1}P'_{d+1,d+1}\left(\tau'_{\mathrm{s}_d},t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)+,$$
$$+\sum_{i=1}^{N}\sum_{r=1}^{d+1}A_{i,r}I'_{i,r,d+1}\left(\tau'_{\mathrm{s}_d},t_{\mathrm{sh}_d}\left(\tau'_{\mathrm{s}_d}\right)\right)\qquad(46)$$

where $P'_{r,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) = P'\left(\left[d-r+1\right]\tau'_{s_d} + t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)$;
$I'_{i,r,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) = I'_i\left(\left[d-r+1\right]\tau'_{s_d} + t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)$.

The next step of 2nd problem solution is the following equality with the following restrictions to be solved

$$\Delta_{\max}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = \max\left|\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right)\right| = \Delta_{\text{pm}} \quad (47)$$

Here the maximization is made over all combination of channel symbols values in transmitted information sequence.

Obviously, to solve this equation it is necessary to analyze the expression $\left|\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right)\right|$ for extrema in the condition $t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right) < \infty$. To do this, we solve the following system using expression (46) and substitution $t_{\text{sh}_d}\left(\tau'_{s_d}\right) = t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)$

$$\left.\begin{aligned}
&\partial\left|\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right)\right| \big/ \partial M_1 = 0\\
&\vdots\\
&\partial\left|\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right)\right| \big/ \partial M_{d+1} = 0\\
&\partial\left|\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right)\right| \big/ \partial A_{1,1} = 0\\
&\vdots\\
&\partial\left|\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right)\right| \big/ \partial A_{N,d+1} = 0
\end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned}
&P'_{1,d+1}\left(\tau'_{s_d}, t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = 0\\
&\vdots\\
&P'_{1,d+1}\left(\tau'_{s_d}, t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = 0\\
&P'_{d,d+1}\left(\tau'_{s_d}, t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = 1\\
&P'_{d+1,d+1}\left(\tau'_{s_d}, t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = 0\\
&I'_{1,1,d+1}\left(\tau'_{s_d}, t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = 0\\
&\vdots\\
&I'_{N,d+1,d+1}\left(\tau'_{s_d}, t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = 0
\end{aligned}\right\}. \quad (48)$$

Analyzing the system (48) we can assert that the extrema exists only for boundary values of SCs due to the fact that considered system has no solutions. Taking into account this fact and the approach presented in the papers [14,15], lets obtain an expression for $t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)$ estimation. Firstly, we will represent $\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)$ as follows:

$$\left|\Delta_{\text{set}}\left(d\tau'_{s_d} + t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right| = \left|S'_+\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) + S'_-\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right|, \quad (49)$$

where

$$S'_+\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) = \sum_{z_1=1}^{d-1} F_+\left(M_{z_1} P'_{z_1,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right) +$$
$$+ F_+\left(M_{d_1}\left[P'_{d,d_1+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) - 1\right]\right)$$
$$+ F_+\left(M_{d+1} P'_{d+1,d_1+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right) + \sum_{i_1=1}^{N}\sum_{r=1}^{d+1} F_+\left(A_{i_1,r} I'_{i_1,r,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right);$$
$$S'_-\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) = \sum_{z_2=1}^{d-1} F_-\left(M_{z_2} P'_{z_2,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right) +$$
$$+ F_-\left(M_{d_2}\left(P'_{d,d_2+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) - 1\right)\right)$$
$$+ F_-\left(M_{d+1} P'_{d+1,d_2+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right) + \sum_{i_2=1}^{N}\sum_{r=1}^{d+1} F_-\left(A_{i_2,r} I'_{i_2,r,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right).$$

Here $d_1 = d_2 = d$.

Considering that the structure of equality (49) repeats the structure of expression (41), we come to the conclusion that the expression for $\Delta_{\max}\left(d\tau'_{s_d} + t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)$ has the following form

$$\Delta_{\max}\left(d\tau'_{s_d} + t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) = M_{\max}\left(\sum_{k=1}^{d-1}\left|P'_{k,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right| + \right.$$
$$+ \left|\left(P'_{d,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right) - 1\right)\right|$$
$$\left. + \left|P'_{d+1,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right|\right) + \sum_{i=1}^{N} A_{\max_i} \sum_{r=1}^{d+1}\left|I_{i,r,d+1}\left(\tau'_{s_d}, t_{\text{sh}_d}\left(\tau'_{s_d}\right)\right)\right|. \quad (50)$$

As a result, the equality $\Delta_{\max}\left(d\tau'_{s_d} + t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right)\right) = \Delta_{\text{pm}}$ can be transformed to the following

$$t_{\text{sh.pm}_d}\left(\tau'_{s_d}\right) = \min T_{\text{sh}}\left(\tau'_{s_d}\right); \quad (51)$$

where $T_{\text{sh}}\left(\tau'_{s_d}\right)$ is the solution of the following equality

$$M_{\max}\left(\sum_{k=1}^{d-1}\left|P'_{k,d+1}\left(\tau'_{s_d}, T_{\text{sh}}\left(\tau'_{s_d}\right)\right)\right| + \right.$$
$$\left.\left|\left(P'_{d,d+1}\left(\tau'_{s_d}, T_{\text{sh}}\left(\tau'_{s_d}\right)\right) - 1\right)\right| + \left|P'_{d+1,d+1}\left(\tau'_{s_d}, T_{\text{sh}}\left(\tau'_{s_d}\right)\right)\right|\right) + . \quad (52)$$
$$+ \sum_{i=1}^{N} A_{\max_i} \sum_{r=1}^{d+1}\left|I_{i,r,d+1}\left(\tau'_{s_d}, T_{\text{sh}}\left(\tau'_{s_d}\right)\right)\right| = \Delta_{\text{pm}}$$

2.3 The solution of 3rd problem

Let's determine the impact of each of the symbols of the sequence on the received one. Obviously, data transmission process, which is described by expression (14), can be qualitatively characterized by the following stages:

1st stage is time interval between two moments, when data transmission starts and when the process at the input of the adaptive amplifier becomes a cyclostationary process.

2nd stage is time interval between two moments, when the process at the input of the adaptive amplifier becomes a cyclostationary process and when transmission ends.

It should be noted that until the considered process becomes cyclostationary, two types of transient processes will be observed at the input of amplifier: the 1st type is associated with the process of information parameter settling during each channel symbol transmission in the sequence, and the 2nd type manifests itself as a process of amplitudes partial pulses settling, which determines the absence of cyclo-stationarity of the observed process, its completion determines the transition to cyclostationarity of the observed process.

Based on the above, it becomes obvious that effective memory estimation should be carried out using the assessment of the stability of the amplitude parameters of partial pulses in the transmitted sequence at the output of the considered FSC. Due to the fact that the main parameter for quality estimation of the system's operation is the capacity, then in accordance with its definition we will consider the transmission process when number of channel symbols in sequence $l \to \infty$.

To solve this problem let us consider the following expression

$$\lim_{l\to\infty}\left[\sum_{j_1=1}^{l}\left[M_{l-j_1+1}P_{l-j_1+1,l}\left(\tau_s\right)+\sum_{i=1}^{N}A_{i,l-j_1+1}I_{i,l-j_1+1,l}\left(\tau_s\right)\right]\right], \quad (53)$$

which was obtained utilizing the expression (37) by using the right side of it and following modifications: $d = l$, $l \to \infty$; $r = l - j_1 + 1$.

According previous results [14,15] the following relation will be valid

$$\lim_{l\to\infty}\left[\sum_{j_1=1}^{l}\left[M_{l-j_1+1}P_{l-j_1+1,l}\left(\tau_s\right)+\sum_{i=1}^{N}A_{i,l-j_1+1}I_{i,l-j_1+1,l}\left(\tau_s\right)\right]\right]\le$$
$$\le\lim_{l\to\infty}\left[\sum_{j_1=1}^{l}\left[\left|M_{l-j_1+1}\right|\left|P_{l-j_1+1,l}\left(\tau_s\right)\right|+\sum_{i=1}^{N}\left|A_{i,l-j_1+1}\right|\left|I_{i,l-j_1+1,l}\left(\tau_s\right)\right|\right]\right]. \quad (54)$$

The following inequality is meet true for any partial sum of first v element of a given series

$$\left|\sum_{j_1=1}^{v}\left[M_{v-j_1+1}P_{v-j_1+1,v}\left(\tau_s\right)+\sum_{i=1}^{N}A_{i,r}I_{i,v-j_1+1,v}\left(\tau_s\right)\right]\right|\le$$
$$\le\sum_{j_1=1}^{v}\left[\left|M_{v-j_1+1}\right|\left|P_{v-j_1+1,v}\left(\tau_s\right)\right|+\sum_{i=1}^{N}\left|A_{i,v-j_1+1}\right|\left|I_{i,v-j_1+1,v}\left(\tau_s\right)\right|\right]. \quad (55).$$

Expressions (54) and (55) reach their greatest values when the following condition is meet true $\forall j_1 : \left|M_{l-j_1+1}\right| = \left|M_{v-j_1+1}\right| = M_{\max}$, $\left|A_{i,l-j_1+1}\right| = \left|A_{i,v-j_1+1}\right| = A_{\max_i}$. So, the estimation rule for dependence effective memory estimation on the symbol duration $\hat{G}\left(\tau_s\right)$ can be represented as follows

$$\hat{G}(\tau_s)=\min\left\{G'(\tau_s):0<\lim_{l\to\infty}\left[\sum_{j_1=1}^{l}\left(M_{\max}\left|P_{l-j_1+1,l}(\tau_s)\right|+\sum_{i=1}^{N}A_{\max_i}\left|I_{i,l-j_1+1,l}(\tau_s)\right|\right)\right]-\right.$$
$$\left.-\sum_{k=1}^{G'(\tau_s)+1}\left(M_{\max}\left|P_{G'(\tau_s)-k+2,G'(\tau_s)+1}(\tau_s)\right|+\sum_{i=1}^{N}A_{\max_i}\left|I_{i,G'(\tau_s)-k+2,G'(\tau_s)+1}(\tau_s)\right|\right)\le\varepsilon\right\}. \quad (56)$$

where ε is the channel symbol amplitude characteristics settling accuracy in the case where the values of channel symbols take the SCs greatest values.

In the case when the considered series can be replaced by its partial sum of $G'\left(\tau_s\right)+1$ channel symbols with accuracy ε at a given value of symbol duration τ_s, the observed output process is cyclostationary one.

Taking into account that the speed of 1[st] type transient processes depends on ratios of channel symbols amplitudes, the further simplification of expression (56) will be done using the following substitutions: $M_{\max}$ on 1 and $A_{\max_i}$ on $\frac{A_{\max_i}}{M_{\max}}$. As result we get

$$\hat{G}(\tau_s)=\min\left\{G'(\tau_s):0<\lim_{l\to\infty}\left[\sum_{j_1=1}^{l}\left(\left|P_{l-j_1+1,l}(\tau_s)\right|+\sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,l-j_1+1,l}(\tau_s)\right|\right)\right]-\right.$$
$$\left.-\sum_{k=1}^{G'(\tau_s)+1}\left(\left|P_{G'(\tau_s)-k+2,G'(\tau_s)+1}(\tau_s)\right|+\sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,G'(\tau_s)-k+2,G'(\tau_s)+1}(\tau_s)\right|\right)\le\varepsilon\right\}. \quad (57)$$

Let's represent the series in expression (57) by its partial sum and remainder, which can be represented as follows:

$$\lim_{l\to\infty}\left[\sum_{j_1=1}^{l}\left|P_{l-j_1+1,l}(\tau_s)\right|+\sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,l-j_1+1,l}(\tau_s)\right|\right]=$$
$$=R_h+\sum_{j_1=1}^{h}\left|P_{l-j_1+1,l}(\tau_s)\right|+\sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,l-j_1+1,l}(\tau_s)\right|, \quad (58)$$

where R_h is series obtained by discarding the first h terms of considered series.

The final expression for estimating $G'\left(\tau_s\right)$ has the following form:

$$\hat{G}(\tau_s)=\min\left\{G'(\tau_s):0<R_h+\sum_{j_1=1}^{h}\left[\left|P_{l-j_1+1,l}(\tau_s)\right|+\sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,l-j_1+1,l}(\tau_s)\right|\right]-\right.$$
$$\left.-\sum_{j_1=1}^{G'(\tau_s)+1}\left[\left|P_{G'(\tau_s)-j_1+2,G'(\tau_s)+1}(\tau_s)\right|+\sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,G'(\tau_s)-j_1+2,G'(\tau_s)+1}(\tau_s)\right|\right]\le\varepsilon\right\}. \quad (59)$$

Interrelation identification between the parameter ε and the accuracy of the resolution time estimate ε_{res} would be considered in the next section of this paper.

3. Algorithm for System Quality Estimation

This section presents algorithms for estimating the resolution time and parameters for assessing the quality of the system's operation based on it.

3.1 Algorithm for estimation resolution time algorithm in the presence of crosstalk

1[st] step. Input is proceeded for the initial parameters which are: SCs configurations; partial pulses $g_{sh_0}(t)$ and $g_{sh_i}(t)$; impulses pulse of InFSC and *i*-IfFSC $h_0(t)$ and $h_i(t)$; BER; variance of noise σ^2; parameter Q_A; resolution time estimation accuracy ε_{res}; gain of adaptive amplifier k_A.

2[nd] step. The permissible settling error Δ_{pm} is calculated according the following expression

$$\Delta_{pm}=Q_A-\Delta=Q_A-k_A\sigma F^{-1}\left(1-BER\times\log_2 n_0\right). \quad (60)$$

3rd step. The largest settling time estimation $t_{\text{set}_{d_r}}$ is calculated by solution the equation (45) with numerical method with accuracy ε_{res} for the fourth symbol $(d_r = 4)$ of PAM-n_0-signal and PAM-n_i-signal modulating information sequence, that follows from the results of paper [14,15,25].

4th step. The value of ε is estimated according to the following rule

$$\begin{aligned} &\varepsilon = \min\left\{\varepsilon'_+ ; \varepsilon'_-\right\}, \\ &\varepsilon'_+ = \min_{T_+}\left|s_{\max}\left(T_+\right) - s_{\max}\left(T_+ + \varepsilon_{\text{res}}\right)\right|, \\ &\varepsilon'_- = \min_{T_-}\left|s_{\max}\left(T_-\right) - s_{\max}\left(T_- - \varepsilon_{\text{res}}\right)\right|. \end{aligned} \quad (61)$$

where $T_+ = \left\{\tau_{\text{w.st}_{d_r,k}}\right\}_{k=1}^{S_{\text{w}}}$; $T_- = \left\{\tau_{\text{w.end}_{d_r,k}}\right\}_{k=1}^{S_{\text{w}}}$;

$$s_{\max}(\,\cdot\,) = \sum_{r=1}^{d_r}\left(\left|P_{r,d_r}(\,\cdot\,)\right| + \sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,r,d_r}(\,\cdot\,)\right|\right).$$

5th step. Parameters for three types ($c = \overline{1;3}$) of majorizing series (see Table 2) are estimated using the following expressions

$$k_c = \frac{\hat{\sigma}_{cC} - \hat{\sigma}_{cD}}{C - D}; \quad (62)$$

$$b_c = \frac{C\hat{\sigma}_{cD} - D\hat{\sigma}_{cC}}{C - D}, \quad (63)$$

where $\hat{\sigma}_{cC}$ and $\hat{\sigma}_{cD}$ are attenuation coefficients for C-th and D-th term of c-th type of majorizing series, respectively. It should be noted that $C, D \in H = \left\{h \middle| h \in \mathbb{N}^*\right\}$, $C > D$ and $C = 10$ and $D = 3$.

The rules for $\hat{\sigma}_{cH}$ calculations presented in table 2 and 3, here H is H-th term of c-th type of majorizing series.

For each type of series, the parameter $\sigma_{ch} = k_c h_c + b_c$ is reconstructed based on the obtained results.

In table 3 the following definitions: S_H is the number of maxima of the following expression for given value of H; $\overset{\circ}{U}\left(\tau_{\text{M}K_i}\right)$ is a punctured neighborhood of the point $\tau_{\text{M}K_i}$; $q > H$ and $q \leq 30$

Table 2

Majorizing series, expressions for estimating attenuation coefficients and the number of elements of the sum to provide a given estimate of the residuals of the series [14, 15]

Parameter	1st type series (c=1)	2nd type series (c=2)	3rd type series (c=3)
Majorizing Series	$\sum_{h=1}^{\infty} u_{1h}(x_s) = \sum_{h=1}^{\infty} \exp(-\sigma_{1h}x_s)$	$\sum_{h=1}^{\infty} u_{2h}(x_s) = \sum_{h=1}^{\infty} \sigma_{2h}x_s \exp(-\sigma_{2h}x_s)$	$\sum_{h=1}^{\infty} u_{3h}(x_s) = \sum_{h=1}^{\infty} (1 + \sigma_{3h}x_s)\exp(-\sigma_{3h}x_s)$
$\hat{h}_c$	$\hat{h}_1 = \left\lceil -\left(\frac{\ln\left(R_{h_1}^1 k_1 x_s\right)}{x_s k_1} + \frac{b_1}{k_1} + 1\right)\right\rceil$	$\hat{h}_2 = \left\lceil -\left[\frac{1 + W_{-1}\left(R_{h_2}^2 k_2 x_s \exp(-2)\right)}{k_2 x_s} + \frac{b_2}{k_2} + 1\right]\right\rceil$	$\hat{h}_3 = \left\lceil -\left[\frac{2 + W_{-1}\left(R_{h_3}^3 k_3 x_s \exp(-2)\right)}{k_3 x_s} + \frac{b_3}{k_3} + 1\right]\right\rceil$
$\hat{\sigma}_{cH}$	$\hat{\sigma}_{1H} = -\frac{\ln E_H}{\tilde{\tau}_s^{(H)}}$	$\hat{\sigma}_{2H} = -\frac{W_{-1}(-E_H)}{\tilde{\tau}_s^{(H)}}$	$\hat{\sigma}_{3H} = -\frac{1 + W_{-1}\left[-E_H \exp(-1)\right]}{\tilde{\tau}_s^{(H)}}$
W_{-1} is omega function with branch -1			

Table 3

Algorithm for k_c, b_c estimation

CFSC impulse response hasn't damped oscillations during settling process
$E_H = \left\lvert P_{q-H+1,q}\left(\tilde{\tau}_{\text{s}}^{(H)}\right)\right\rvert + \sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left\lvert I_{i,q-H+1,q}\left(\tilde{\tau}_{\text{s}}^{(H)}\right)\right\rvert = \min_{\substack{\tau'_{\text{s}} \to \max \\ \tau'_{\text{s}} \in \left[\tau_{\text{w.st}_{d_r,1}};3\tau_{\text{w.end}_{d_r,1}}\right]}} \times \left\{\left\lvert P_{q-H+1,q}\left(\tau'_{\text{s}}\right)\right\rvert + \sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left\lvert I_{i,q-H+1,q}\left(\tau'_{\text{s}}\right)\right\rvert : \left\lvert P_{q-H+1,q}\left(\tau'_{\text{s}}\right)\right\rvert + \sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left\lvert I_{i,q-H+1,q}\left(\tau'_{\text{s}}\right)\right\rvert \neq 0\right\}$
CFSC impulse response has damped oscillations during settling process
$E_H = \left\lvert P_{q-H+1,q}\left(\tilde{\tau}_{\text{s}}^{(H)}\right)\right\rvert + \sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left\lvert I_{i,q-H+1,q}\left(\tilde{\tau}_{\text{s}}^{(H)}\right)\right\rvert$; $\tilde{\tau}_{\text{s}}^{(H)} = \max T'_H$; $T'_H = \left\{\begin{array}{l}\tau_{\text{M}K_i} \in \tau'_{\text{s}} \,\middle\vert\, \tau'_{\text{s}} = \left[\tau_{\text{w.st}_{d_r,1}}, 3\tau_{\text{w.end}_{d_r,\text{Sw}}}\right], \\ \left(\forall \tau''_{\text{s}} \in \overset{\circ}{U}\left(\tau_{\text{M}K_i}\right)\right) \wedge \left(\tau''_{\text{s}} \subset \tau'_{\text{s}}\right)\end{array}\right.$ $\left(\left\lvert P_{q-H+1,q}\left(\tau''_{\text{s}}\right)\right\rvert + \sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left\lvert I_{i,q-H+1,q}\left(\tau''_{\text{s}}\right)\right\rvert \leq \left(\left\lvert P_{q-H+1,q}\left(\tau_{\text{M}K_i}\right)\right\rvert + \sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left\lvert I_{i,q-H+1,q}\left(\tau_{\text{M}K_i}\right)\right\rvert\right); K_i = \overline{1, S_H}\right)\Big\}$

6th step. The selection optimal series type c_{opt} is made using following expression, which is obtained using the results of papers [16, 21]

$$c_{\text{opt}}=\begin{cases}\arg\min\limits_{c\in\overline{1;3}}\sum\limits_{q=1}^{3}\delta_{qc}h'_q \text{ if } \left(h'_c=h'_{c_1}\right)\wedge\left(c=c_1\right),\\ \arg\min\limits_{c\in I'}\left(\max\limits_{T_{\text{comp}}}\left\{u_{ch'_c}\left(T_{\text{comp}}\right)-\left[\left|P_{l'-h'_c+1,l'}\left(T_{\text{comp}}\right)\right|+\right.\right.\right.\\ \left.+\sum\limits_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,l'-h'_c+1,l'}\left(T_{\text{comp}}\right)\right|\right]u_{ch'_c}\left(T_{\text{comp}}\right)-\\ \left.\left.-\left[\left|P_{l'-h'_c+1,l'}\left(T_{\text{comp}}\right)\right|+\sum\limits_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,l'-h'_c+1,l'}\left(T_{\text{comp}}\right)\right|\right]>0\right\}\right)\\ \text{if } \left(h'_c=h'_{c_1}\right)\wedge\left(c\neq c_1\right).;\end{cases} \tag{64}$$

where δ_{qc} is Kronecker delta; $T_{\text{comp}}\in\left[\tau_{\text{w.st}_{d_r,1}};\tilde{\tau}_{\text{s}}^{(d_r)}\right)$; $I'=\arg\min\limits_{c.\in\overline{1;3}}\sum_{q=1}^{3}\delta_{\text{qc}}h'_q$; h'_q is calculated using the following expression

$$L_{\text{res}}=\left\{h'_c\middle|\forall q\in\left[h'_c;l'\right)\left(u_{cq}\left(T_{\text{comp}}\right)-\left[\left|P_{l'-q+1,l'}\left(T_{\text{comp}}\right)\right|+\sum_{i=1}^{N}\frac{A_{\max_i}}{M_{\max}}\left|I_{i,l'-q+1,l'}\left(T_{\text{comp}}\right)\right|\right]\geq 0\right);c=\overline{1,3}\right\}. \tag{65}$$

Here u_{cq} is q-th term c-th type series. $l'\leq 30$;

7th step. $\hat{G}\left(\tau_{\text{s}}\right)$ is estimated utilizing (59) with assumption $R_h=R_{\hat{h}_{c_{\text{opt}}}}^{c_{\text{opt}}}=Q_\varepsilon\varepsilon$ and $h=\hat{h}_{c_{\text{opt}}}$. Here $Q_\varepsilon\in\left[0,1;0,01\right]$, $\hat{h}_{c_{\text{opt}}}$ is calculacted based on the given value $R_{\hat{h}_{c_{\text{opt}}}}^{c_{\text{opt}}}$ using the expressions for $\hat{h}_{\text{c}}$, presented in Table 2.

Then, the effective memory is estimated using the following expression

$$G=\begin{cases}\hat{G}\left[\min\left(t_{\text{set}_{d_r}}\right)\right]-1, & \text{if } \hat{G}\left[\min\left(t_{\text{set}_{d_r}}\right)\right]-1\geq d_r,\\ d_r-1, & \text{if } \hat{G}\left[\min\left(t_{\text{set}_{d_r}}\right)\right]-1<d_r.\end{cases} \tag{66}$$

8th step. Based on the effective memory estimation G, the resolution time $t_{res}=t_{\text{set}_{G+1}}$ and lower bound capacity estimation are calculated by solution the equation (45) and utilizing expression (35) and (36) for given values of BER and G.

3.2 Algorithm system quality parameters estimations in the presence of crosstalk

For the initial parameters which are: SCs configurations; given value of BER; variance of noise σ^2; parameter Q_A; gain of adaptive amplifier k_{A}; resolution time t_{res}; effective memory G; feducial symbol sampler instability p the algorithm for symbol synchronization parameters estimation can be presented in the form of the following steps:

1st step. For channel symbol duration $\tau_{\text{s}}=\tau_{\text{w.st}_1}$ permissible time shift $T_{\text{sh.pm}}$ is calculated using expressions (26) or (27) and (51), solving equality (52).

2nd step. For channel symbol duration $\tau_{\text{s}}=\tau_{\text{w.st}_1}$ optimal time shift ΔT_{opt} and ΔT_{opt_d} are calculated using expressions (23) and (24), respectively, using expression (50) and $T_{\text{sh.pm}}$.

3rd step. The largest spread of optimal time shifts for retrieving information δ_{spr} using expression (28) is made.

4th step. For given value of gain of adaptive amplifier k_{A} The attenuation SA caused by the incomplete amplitude settling of the partial signal pulse due its short duration and transient processes is estimated utilizing expression (34).

5th step. Min SNR and Min $SNR(p)$ are estimated using expressions (32) and (33).

Conclusion

In this paper, a method with ultra-low computational complexity has been developed for low bound capacity estimation and signal integrity for broadband telecommunication system with PAM-n-signals functioning in the presence of crosstalk. The number of equations that need to be solved to determine the resolution time does not depend on the number of discrete states in the signal constellations and linearly depends on the number of crosstalk sources and the effective channel memory.

The main limitation of developed method is that it can only be used for the case when the durations of channel symbols of the information and interfering signals are the same and the start times of their transmission coincide.

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ПРИМЕНЕНИЕ ТЕОРИИ РАЗРЕШАЮЩЕГО ВРЕМЕНИ ДЛЯ РАЗРАБОТКИ И ОЦЕНКИ КАЧЕСТВА ШИРОКОПОЛОСНЫХ СИСТЕМ ПЕРЕДАЧИ ИНФОРМАЦИИ НА ОСНОВЕ ДВУХПОЛЯРНЫХ АИМ-N-СИГНАЛОВ В УСЛОВИЯХ ПЕРЕКРЁСТНЫХ ПОМЕХ

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Аннотация
В статье представлен новый алгоритм с линейной временной сложностью для оценки разрешающего времени и нижней границы пропускной способности частотно-селективного канала связи, зависящей только от значения эффективной памяти канала и числа источников перекрестных помех, при использовании линейного приемника и двухполярных многопозиционных амплитудно-импульсных сигналов. Основным ограничением метода является то, что он применим только для случая, когда длительности канальных символов информационного и мешающего сигналов одинаковы, а моменты времени начала их передачи совпадают. С практической точки зрения данный метод может быть использован при анализе высокоскоростных проводных интерфейсов передачи данных, в которых информация передается одновременно и параллельно по нескольким линиям связи, расположенным достаточно близко друг к другу. Ключевыми особенностями данного метода являются: 1) постоянное число уравнений, равное 1, необходимое для оценки наибольшего времени установления; 2) линейная зависимость числа членов уравнения от эффективной памяти и количества источников перекрестных помех; 3) новая, более точная процедура оценки эффективной памяти канала, позволяющая одновременно определять время разрешения и пропускную способность; 4) новый набор оценок, позволяющий оценить требования к подсистемам синхронизации символов; 5) новый метод оценки требуемого минимального отношения сигнал/шум.

Ключевые слова: *МСИ, разрешающее время, пропускная способность, теория разрешающего времени, АИМ-n-сигналы, алгоритм с линейной вычислительной сложностью, перекрёстные помехи*

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