ESTIMATION THE NOISE IMMUNITY OF CONTINUOUS PHASE MODULATION SIGNALS WITH FULL RESPONSE BASED SIMULATION MODEL

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The problem of spectral load of communication channels is now becoming urgent, which determines the efforts of the world's leading tech communication companies to search for perspective signals. These signals must ensure compliance with international standards governing the use of frequency bands and the permissible level of out-of-band emissions and the required noise immunity of information transmission. The article deals with the noise immunity of continuous phase modulation signals research. A preliminary estimation is carried out by calculating the Euclidean distances between signals. The estimates of the noise immunity were obtained experimentally. The results of simulation are presented for fixed value of the signal to noise ratio for signals. Modulation indices and phase pulse were taken into account. The results of the study make it possible to emphasize the modulation indices for CPM FR signals, having more noise immunity then the known BPSK and MSK signals. Increasing the noise immunity becomes possible due to intersymbol phase communication. These signals are received according to the Viterbi algorithm. The results of the study show the values of the modulation indices to be of practical interest.

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Introduction

In modern digital communication systems (*DCS*) noise immunity is determined by the *BEP* (*bit error probability*) value. In general the *BEP* value depends on the bit energy to the noise power spectral density at the input of the receiver E_b/N_0, the parameters of communication channel, the modulation method, the number of levels use in a particular modulation method, the signal reception method.

The most attractive to research are *CPM* (*continuous phase modulation*) signals. Due to the constancy of their envelope, a higher rate of decrease in power spectral density is provided compared to *BPSK* (*binary phase shift modulation*). Moreover, the use of *CPM* signals when choosing certain parameters and an effective algorithm allows achieving better noise immunity than using *BPSK* signals. The increase of the noise immunity in this case is explained by the fact that many different symbols differ not only in frequency, but also in the initial phase. Hence, adjacent symbols *CPM* signals have an intersymbol phase relationship (*IPR*), since the initial phase of any symbol depends on several previous information symbols [2,5,6].

This study focuses on *CPM FR* (*full response*) signals. In this case one symbol of the information sequence is transmitted during the symbol time. However, *CPM PR* (*partial response*) signals can be used to conserve frequency resources and improve power performance. In this case, the information transfer rate remains the same and the symbol time is increased by a factor L, where L is an ineteger [1,7,10].

This article is devoted to the estimation of the noise immunity of binary *CPM* signals and to the study of the influence of the parameters of these signals on the noise immunity at fixed values of E_b/N_0.

Potential noise immunity analysis of the *CPM* signals

In general *CPM* signal can be written:

$$s(t) = \sqrt{\frac{2E}{T_s}} \cos\left(\omega t + \varphi(t:I_n)\right) \quad 0 \le t \le NT, \qquad (1)$$

where E is a bit energy;

$\varphi(t:I_n)$ – function is used to describe the phase trajectory over a given time interval [7,9,10,12]:

$$(t:I_n) = \theta_n + 2\pi h q(t - nT) \quad 0 \le t \le NT, \qquad (2)$$

I_n – data symbols, each taking one of the values ± 1;

θ_n determines the depth of *IPR* depending on the modulation index h used in the *DCS*:

$$\theta_n = \pi h \sum_{\kappa=-\infty}^{n-1} I_k \qquad (3)$$

h – modulation index. This parameter determines the spectral properties and energy efficiency of *CPM* signals.

In modern *DCS MSK* (*minimum shift keying*) is widely used. It is a *CPM* signal with modulation index 0.5.

$q(t)$ – phase pulse (*PP*) [2,5,6]. It defines the law of phase change and the level of out-of-band emissions:

$$q(t) = \int_0^t g(t)dt \qquad 0 \le t \le T, \qquad (4)$$

Formulas (5) determine the *PP* for a rectangular baseband pulse (1*REC*) and a pulse with non-linear phase law of the «raised cosine» class (1*RC*). This class is used to reduce the level of out-of-band emissions:

$$q_{1REC}(t) = \begin{cases} 0, (t < 0), \\ \frac{t}{2T}, (0 \le t < T) \\ \frac{1}{2}, (t \ge T) \end{cases}$$

$$q_{1RC}(t) = \begin{cases} 0, (t < 0), \\ \frac{1}{4}\left(1 - \cos\frac{\pi t}{T}\right), (0 \le t < T) \\ \frac{1}{2}, (t \ge T) \end{cases} \qquad (5)$$

In according to the geometric interpretation of signal theory the potential noise immunity is determined by the minimum d^2_{min} between the ends of two signal vectors (two phase trajectories). For the binary case d^2_{min} is defined:

$$d^2_{min} = \lim_{N\to\infty} \min_{i,j} \frac{1}{T} \int_0^{NT} 1 - \cos\left(\varphi\left((t:I_i) - \varphi(t:I_j)\right)\right) dt, \quad (6)$$

where N – the number of symbol intervals involved in deciding on the value of the first symbol of the transmitted information sequence. The values obtained by the formula (6) are normalized to the $2E$ value [2,6,7]

Of particular interest is the upper bound d^2_B of the Euclidean distances. The upper bound determines the maximum for a given modulation index energy difference between the two signals before the first merging ($d^2_{min}=0$) of the phase trajectories [2,6,7]. It is used to determine the potential noise immunity of *CPM* signals and is a critical value. The upper bound value for 1*REC* for corresponding h is defined by (7):

$$d^2_B(h) = 2\left(1 - \frac{\sin(2\pi h)}{2\pi h}\right), \qquad (7)$$

When calculating the minimum d^2_{min}, all values exceeding the upper bound d^2_B are not taken into account in the future. The algorithm for calculating the dependence of the minimum Euclidean distances on the modulation index is considered in detail in [2,4].

Figure 1 is a good illustration of d^2_{min} versus h for a given N (bit decision intervals) and phase pulse 1*REC*, 1*RC*.
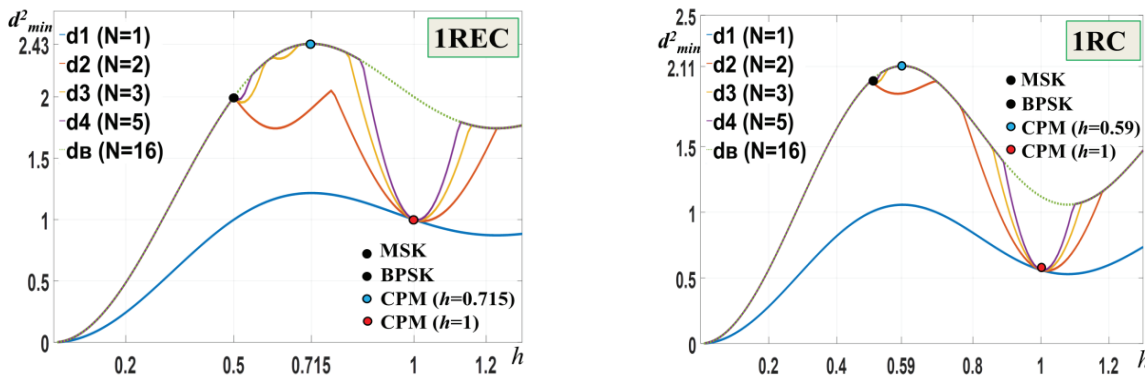

Fig. 1. Euclidean distances versus modulation index for binary *CPM FR*

Analysis of d^2_{min} allows selecting the values of h, the use of which makes it possible to increase the noise immunity of *CPM* signals compared to *BPSK* and *MSK*. It also makes it possible to emphasize the values of the modulation indices h, the use of which is impractical. However, *BEP* estimates are needed to decide whether a specific modulation index in DCS is appropriate. *BEP* estimates determine the DCS immunity at fixed values of E_b/N_0.

Experimental estimation of *CPM* signals noise immunity. Study of the influence of modulation index and phase pulse shape on the *BEP* value.

Determination of the dependence of *BEP* on E_b/N_0 for *CPM* signals is closely connected with finding the dimension of the signal space, determining the spatial packing of signals vectors in it. Then it's necessary to divide the signal space into solution regions and to enumerate all variants of errors. It's extremely difficult to obtain an analytical solution to this problem. For this reason, the decision was made to abandon the exact calculation of the *BEP*. This difficulty is overcome in practice by calculating the upper limit of this probability.

In [2,6] it is proposed to estimate the probability of error using the combined upper limit:

$$BEP \leq \frac{1}{m^{N-1}} \sum_k^N \sum_l^N 1 - \Phi[\frac{D(P^1_{\alpha,N};P^2_{\beta,N})}{\sqrt{2N_0}}], \qquad (8)$$

$D(P^1_{\alpha,N};P^2_{\beta,N})$ – Euclidean distance between information sequences P^1 and P^2 of length N with a different first symbol;

$\Phi(x)$ – tabulated Crump function:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{z^2}{2}} dz = \frac{1}{2} + \frac{1}{2} er f\left(\frac{x}{\sqrt{2}}\right), \qquad (9)$$

However, the combined upper limit turns out to be too overestimated and can undeservedly exclude *CPM* signals with a sufficiency high but difficult to calculate noise immunity out of consideration. Thus, the simplified method for evaluating the noise immunity isn't suitable for analyzing the noise immunity of *CPM* signals.

Therefore, a more accurate determination of the dependence of *BEP* on E_b/N_0 for *CPM* signals is possible through simulation only. For this, the authors developed a simulation model of a radio channel with *CPM* using the *Matlab* 2016b trial package [3]. Decoding of the received sequences is carried out according to the Viterbi algorithm, which allows taking into account the *IPR*.

The number of symbols of the transmitted sequence is chosen based on the authenticity requirements (confidence level) of obtaining estimates of the noise immunity of *DCS* with *CPM*. According to the theory of mathematical statistics, the value V of the number of transmitted symbols is a function of the error probability per bit and confidence level.

BER (*bit error rate*) is taken as an estimate of the *BEP*. *BER* is the rate m/V of the occurrence of an event consisting in the erroneous reception of m symbols from the total number of V received symbols [8, 11].

The following to the random reasons, the *BER* estimate may generally differ from *BEP*. Values are introduced: ε – the accuracy of the estimate и α – the authenticity of estimate:

$$|BEP - BER| < \varepsilon \text{ и } \alpha = P\{BEP - BER| < \varepsilon\} \qquad (10)$$

In this case, the number V of transmitted symbols required to obtain an estimate with accuracy ε and authenticity α is given by:

$$V = t_\alpha^2 \frac{BER(1-BER)}{\varepsilon^2} \qquad (11)$$

t_α Student's coefficient. With a confidence level of 0.95 and a sufficiently large value of the transmitted symbols V the Student's coefficient will be 0.95.

Table 1 shows the data that make it possible to determine the minimum number of symbols V f or various values of the *BEP*.

Table 1

The number V of symbols needed to estimate the *BEP*

P_b	ε=0.05* P_b	ε=0.1* P_b	ε=0.15* P_b	ε=0.2* P_b
10^{-1}	$1.38*10^4$	$3.46*10^3$	$1.54*10^3$	$8.65*10^2$
10^{-2}	$1.52*10^5$	$3.8*10^4$	$1.69*10^4$	$9.51*10^3$
10^{-3}	$1.54*10^6$	$3.84*10^5$	$1.71*10^5$	$9.59*10^4$
10^{-4}	$1.54*10^7$	$3.84*10^6$	$1.71*10^6$	$9.6*10^5$
10^{-5}	$1.54*10^8$	$3.84*10^7$	$1.71*10^7$	$9.6*10^6$

The estimates of the *BEP* (*BER*) obtained in the course of simulation modeling for *CPM FR* signals with different modulation indices h at E_b/N_0=8.4 dB are shown at figure 2.

The reliability of the results is ensured by the coincidence of the values of E_b/N_0 of the known radio signals obtained during the simulation with their a priori known values. So, for *BPSK* and *MSK* signals having the same *BEP* value at a fixed value of E_b/N_0, to ensure *BEP* = 10^{-4} the value E_b/N_0 = 8.4 [7,10].
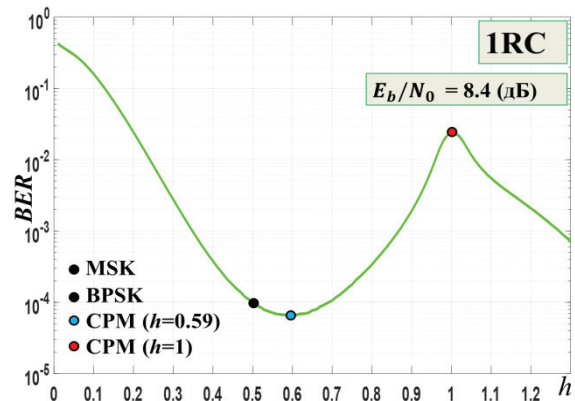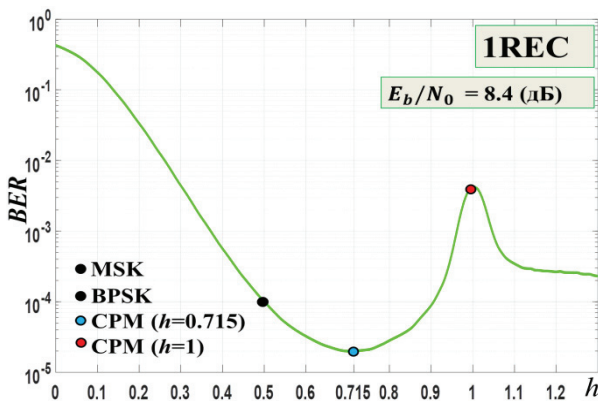

Fig. 2. Dependence of the *BER* of *CPM FR* signals on the modulation index h at a fixed value of E_b/N_0=8.4 dB

The adequacy of the results obtained is confirmed by the fact that results of simulation and theoretical data correspond to each other. For a 1*REC* phase pulse, the maximum d^2_{min}, and, consequently, the maximum noise immunity is achieved for the modulation index h=0.715 at N=3 and reaches the value d^2_{min}=2.43. This result is the same as those obtained in the studies [2, 5]. Figure 2 shows that for h=0.715 there is a minimum value of the *BER*. In the range of h values from 0.5 to 0.715 an increase in the values of Euclidean distance is observed (see Figure 1). Figure 2 illustrates that for the specified range of values of h there is a decrease in the values of the *BER*. A further increase of h leads to a decrease in the values d^2_{min}, and an increase in the *BER* to the value of modulation index h=1. For this h the maximum value of the *BER* is observed in the range of modulation indices from 0.5 to 1. This fact is explained by the fact that when h=1 there is no *IPR* in the *CPM* signals [2,6,7].

Similarly, for the 1*RC* phase pulse in the modulation index range from 0.5 to 0.6, it can be seen that the corresponding Euclidean distances $d^2_{в}$ increase, but starting from 0.6 and up to 1, a rather sharp decrease in the values of the Euclidean distances is observed. In the specified range, there are significant differences between the 1*RC* from the 1*REC*, which demonstrates an increase in $d^2_{в}$ up to the value h=0.715. Further fall of $d^2_{в}$ to h=1 is clearly less pronounced than 1*RC* phase pulse. When comparing the above dependence, it can be seen that both *CPM FR* signals have the same minimum Euclidean distances at the modulation index h=0.5 (*MSK*). With an increase in the value of h, the largest Euclidean distance has a signal with 1*REC* phase pulse.

Table 2 shows the values of the upper bound of the Euclidean distances $d^2_{в}$ for *CPM FR* signals of which use is of practical interest, the values of E_b/N_0 at which the *BEP* = 10^{-4} is provided, and error reduction factor k in comparison with the used *BPSK, MSK*.

Table2

Comparing *CPM FR* to *BPSK, MSK* signals

Modulation index, h	Phase pulse, $q(t)$	The upper bound, $d^2_{в}$	E_b/N_0, (dB)	Error reduction factor, k
0.66	1REC	2.40	7.56	4.11
0.715	1REC	2.43	7.51	4.85
0.55	1RC	2.09	8.09	1.25
0.59	1RC	2.11	8.05	1.55

As can be seen from table 2 the use of *CPM FR* signals with the indicated modulation indices h can significantly increase the noise immunity of *DCS* in comparison with the used *BPSK* and *MSK* signals.

Estimation of the noise immunity *CPM FR* signals

It is proposed to pre-select signals for *DCS* using the d^2_{min} between signals, the final selection of signals using the *BEP*.

Estimation includes 5 points:

1. De nition of input data: choice of the value of E_b/N_0 for which is necessary to obtain an estimate of the noise immunity, selection of phase pulse shape, bit rate, selection modulation index h range, the values of confidence level and authenticity during simulation.

2. D ermination of the potential noise immunity of *CPM FR* signals by calculating the Euclidean distances according to (6) within the specified range of the modulation index h.

3. hoice of the values of the modulation indices h with greater Euclidean distances than those of the used *BPSK* and *MSK* signal*s*.

4. mulation modeling for given input data and modulation index values defined in item 3.

5. nalysis of the results obtained. Checking the correspondence of theoretical data to the research results. Taking into account the complexity of the hardware and software implementation of signal shaping devices, the issue of the expediency of using one or another modulation index h of the phase pulse shape in the designed *DCS* is being resolved.

Conclusion

For a preliminary estimation of the noise immunuty of *CPM FR* signals using the geometric interpretation of the signal theory, an analysis of the potential noise immunity was carried out. Analytical dependences on the modulation indices h are obtained for 2 types of radio signals of the studied class. Analytically the presence of a number of indices by modulation has been established, the use of which in the designed *DCS* makes it possible to achieve more nise immunity than when using the known *BPSK* and *MSK* signals.

An estimate of *BEP* for a given class of signals was obtained using a simulation model. It was found that the results of simulation and theoretical data are consistent with each other. Estimates of the *BEP* are obtained for a *CPM FR* signal with different modulation indices h at a fixed value E_b/N_0=8.4 dB. Better noise immunity in comparison with non-linear laws of phase change 1*RC* has 1*REC* phase pulse.

Thus, the results of the study make it possible to emphasize the modulation indices h for *CPM FR* signals, having more noise immunity then the known *BPSK* and *MSK* signals. In addition, the results become possible to estimate the *BEP* for *CPM FR* signals at fixed values of E_b/N_0 and a selected phase pulse shape with a given accuracy.

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ОЦЕНКА ПОМЕХОУСТОЙЧИВОСТИ ЧАСТОТНО-МОДУЛИРОВАННЫХ СИГНАЛОВ С НЕПРЕРЫВНОЙ ФАЗОЙ И ПОЛНЫМ ОТКЛИКОМ НА ОСНОВЕ ИМИТАЦИОННОГО МОДЕЛИРОВАНИЯ

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Аннотация

Статья посвящена исследованию помехоустойчивости частотно-модулированных сигналов с непрерывной фазой и полным откликом. Предварительная оценка помехоустойчивости проведена путем расчета минимальных евклидовых расстояний между сигналами. Оценки помехоустойчивости исследуемых сигналов получены экспериментально. Представлены результаты имитационного моделирования при фиксированном значении отношения энергии бита к спектральной плотности мощности шума для исследуемых сигналов с заданными индексами модуляции и формой фазового импульса. Результаты исследования позволяют выделить сигналы, обладающие помехоустойчивостью лучшей, чем используемые в настоящее время сигналы с BPSK и MSK, а также получить для них оценки вероятности ошибки на бит с заданной точностью. Повышение помехоустойчивости становится возможным за счет межсимвольной фазовой связи, возникающей вследствие отсутствия разрывов фазы несущего колебания на границах символьных интервалов, и ее учета при декодировании принимаемой последовательности с использованием алгоритма Витерби.

Ключевые слова: помехоустойчивость, частотно-модулированные сигналы с непрерывной фазой и полным откликом, евклидово расстояние, индекс модуляции, вероятность ошибки на бит.

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