MATHEMATICAL MODELS FOR TRAFFIC FLOWS ON HIGHWAYS WITH INTERSECTIONS AND JUNCTIONS

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Mathematical models of motor traffic flow on highway sections on highway sections near intersections or flow segregation sections are considered. In these models, the particles corresponding to motor vehicles move according to probabilistic rules along a cellular field that moves at a constant speed in the direction coinciding with the direction of movement of the particles. A cell field consists of sequences of cells. Each such sequence corresponds to a lane on the highway. The time scale in the model is discrete or continuous. The model is a dynamic system with a discrete state space and discrete or continuous time. The mathematical description of the model can also be presented in terms of a cellular automaton or a random process with prohibitions. At any given time, there is no more than one particle in each cell. With each movement, the particle either moves one cell in the direction of movement, or moves to the next lane, or remains in place. The speed of the traffic flow on the highway section corresponds to the sum of the set speed of the cell field and the average speed of the particles relative to the field. The studied characteristics are the speed of the traffic flow, its intensity and the probability of successful rebuilding of the vehicle on the considered section of the highway. When setting the parameters of the model, data from measurements of the characteristics of traffic flows on highways are used. Analytical approaches have been developed to evaluate the studied characteristics. Computer programs have been created to implement the developed calculation algorithms. The results of calculations are given.

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Introduction

This paper discusses the application of mathematical modeling in the problems of traffic management, such as the organization of the right turn from the main road to the secondary road at unregulated intersections with pedestrian crossings, the choice of the moment when the vehicle changes lanes in the area of segregation of traffic flow, the organization of traffic on the stretch between intersections.

The issues of developing methods for assessing the quality of motor vehicles for a number of properties were considered in [1].

The study of the characteristics of road traffic flows using mathematical and simulation modeling is related to the issues of road safety and environmental safety [2]-[4].

In mathematical models of motor traffic flow, referred to the class of microscopic models, the movement of vehicles is described by the movement of particles through a cellular field (lattice). Each vehicle corresponds to one particle. The field is divided into cells (cells) that form infinite or closed sequences, with each such sequence corresponding to a lane. Models of this class can be interpreted in terms of cellular automata [5] or random processes with prohibitions [6]. For the simplest single-band models, analytical results were obtained, for example, in [7]-[13]. The characteristics of multiband motion are investigated using simulation modeling, or in some works, simplifying assumptions are made in [14]-[16] when constructing the model in order to obtain analytical results, for example, in [16] it is assumed that particles move on a toroidal lattice. In [17]-[24], mathematical models of motor traffic flow in sections of multi-lane traffic near road intersections are considered. These works use the deterministic-stochastic approach developed by A.P. Buslaev [25], according to which the flow velocity is represented as the sum of the deterministic component, which corresponds to the constant velocity of movement of the cellular field set in the model and the stochastic component corresponding to the average velocity of particles relative to the field. The deterministic-stochastic approach was also used in modeling single-lane traffic, for example, in [13].

1. Models of realignments of motor vehicles on the highway section

In [17], a mathematical model of traffic on a multi-lane road in a stream is considered, where it is necessary to change lanes from one extreme lane to another. It is necessary to estimate the capacity of the stage depending on its parameters. A method has been developed for estimating the throughput of the stage depending on its parameters. As an example, the results of calculations for specific parameter values are given. This kind of problems are encountered when managing traffic with saturated traffic flows on radial-ring road networks.

In [17] an example of calculating the probability of successful rebuilding of a motor vehicle on a highway stretch is given.

In the considered [17] model, a particle moves on a rectangular lattice, Fig. 1. The location of particle is characterized by the pair of coordinates (x,y), where the coordinate $x = 0,1,\ldots,n$ corresponds to the number of the lane, and the coordinate $y=0, d, 2d, \ldots$ characterizes the location of a particle in the lane. The length of each zone equals l cells, and $L = 2l$ is the length of the segment. Suppose, at initial time 0, a particle is at the point (0,0).

The coordinates of a particle can change at time $i\Delta t$, $i=0,1,2,\ldots$. If, at time $(i+1)\Delta t$, a particle is at the point (k, ld), $k<n$, then, at time $(i+1)\Delta$, with probability p_1 will be at the point $(k+1,(l+1)d)$, the particle will be at the point $(k,(l+1)d)$ with the probability p_2, the particle will be at the point $(k+1,(l+1)d)$ with the probability p_3, and the particle will continue to be at the point (k, ld), $k<n$, with the probability p_0, $p_1+p_2+p_3+p_0=1$. The value d is assumed to be approximately the dynamical dimension, i.e., the sum of the vehicle length and the distance between vehicles satisfying the safety conditions. It is assumed that the dynamical dimension is represented approximately by a quadratically polynomial of the traffic flow speed. The model is based on the deterministic-stochastic approach for modeling transport flows.

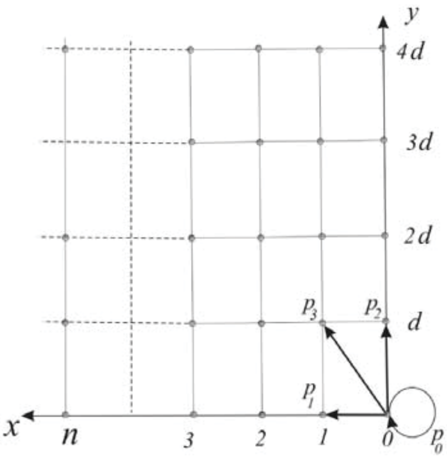

Fig. 1. Movement of a particle on the cellular field [17]

Suppose ξ is a random value equal to the particle ordinate at the moment at that the abscissa reaches the value k, and τ_k is the related moment, $k=1,\ldots,n$. In [17], the following formulas are obtained for the expectation $\mathrm{M}\xi_k$, $\mathrm{M}\tau_k$ and dispersion $\mathrm{D}\xi_k$, $\mathrm{D}\tau_k$ of ξ_k and τ_k:

$$\mathrm{M}\xi_k = \frac{p_2+p_3}{p_1+p_3} kd, \tag{1}$$

$$\mathrm{M}\tau_k = \frac{k\Delta t}{p_1+p_3}, \tag{2}$$

$$\mathrm{D}\xi_k = kd^2 \left[\frac{p_2+p_3}{p_1+p_3} d^2 + \frac{(p_1+p_3)}{(p_1+p_3):2} \right], \tag{3}$$

$$\mathrm{D}\tau_k = k \frac{(p_2+p_0)(\Delta t)^2}{(p_1+p_3)^2}. \tag{4}$$

In [17], the generating function for joint distribution of ξ_k, τ_k is obtained:

$$\pi_k(z_1, z_2) = \left(\frac{p_1 z_2 + p_3 z_1 z_2}{1 - p_0 z_2 - p_2 z_1 z_2} \right)^k. \tag{5}$$

Assume that the probability that a point of the lattice is occupied equals r independent of the other points states. The attempt of a particle is realized only if the cell to that the particle moves is vacant. In accordance with this assumption, the probabilities p_1, p_2, p_3, p_4 are computed. Using the deterministic-stochastic approach and (1)-(5), one may compute some main characteristics of movement.

The following example is considered in [17]. Suppose the highway contains m sections, and the length of a highway segment is equal to L m. The deterministic component of the traffic flow velocity is equal to v m/sec. Each lane is a sequence of cells, and the number of these cells is equal to $mL/d(v)$, where $d(v)=c_0+c_1v+c_2v^2$, c_0, c_1, c_2 are some constants, $d(v)$ (in meters) is the dynamical dimension.

One may compute the traffic intensity q 1/sec may compute using the formula

$$q=\frac{(1+n)r}{d(v)}\left[v+\lambda p(1-r)d(v)\right],$$

where λ, p satisfy the equations

$$p_1+p_3=\lambda\Delta t(1-r)$$
$$p_1+p_2+p_3=\lambda\Delta t(1+pr).$$

The value q is computed under the assumption that

$$n\left[pd(v)+\frac{v}{\lambda(1-r)}\right]=L.$$

Suppose

$$\rho=r/d(v)=C/[(n+1)mL,$$
$$\rho=r/d(v)=C/[(n+1)mL]$$

$\rho=1/m$ is the traffic flow density, $S=mL$. In Fig. 2, the dependence on the flow intensity on the flow density are shown.

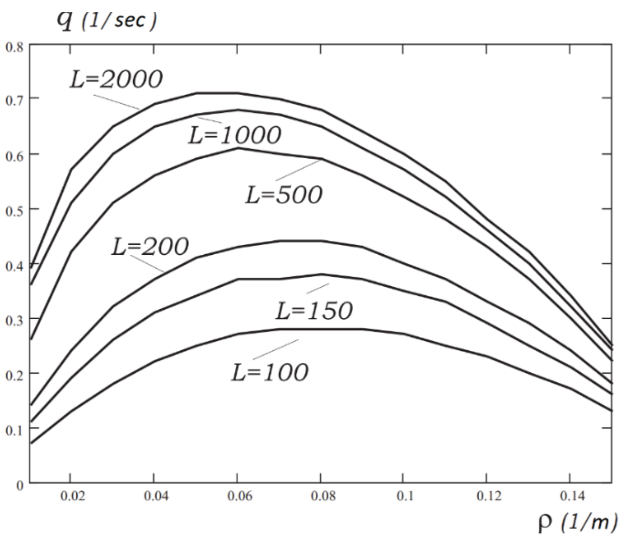

Fig. 2. The dependence of the flow intensity on the density [17]

The paper [22] develops an approach to choose a place to begin a change of lane intending to make a turn and choose the traffic lane phases duration. These durations depend on the time of day. The methodology is developed on the base of a transport mathematical models of the transport flow on a section of a highway and on the base of measurements on highways. The results of these measurements were compared with data obtained in the measurements. The model allows to compute the traffic intensities for a section highway. The distribution of these intensities has been computed for a section of Leingradskiy prospect. The model is also based on the deterministic-stochastic approach. Using the deterministic-stochastic approach, one may obtain more accurate values of the traffic intensities.

2. Models of traffic flow segregation

Freeway interchange influence areas are very important elements of highways, which often determine the highway capacity, (Fig. 3).

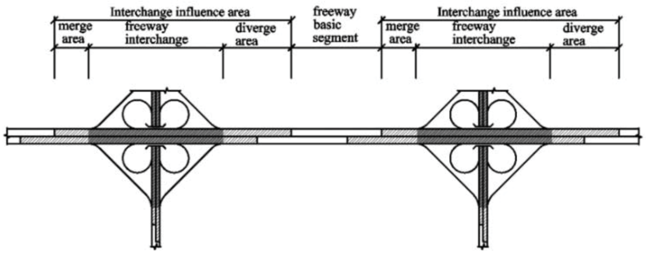

Fig. 3. Freeway interchange influence areas

The paper [20] proposes the following model. The particles of two types move on K-lane section, which is divided into M segments. The mth segment contains cells of size (length) d_m. The location of a cell is characterized by coordinates (x,y), where x is the lane index, and y characterizes the location of the cell in the lane. The values of y satisfy the condition $(m-1)l<y\leq ml$. There are two types of particles. Particles of the first type do not tend to transit to another lane. Any particle of the second type tends to transit successively to the Kth lane.

The particles of this type correspond to vehicles intending to move along another road after passing the section. The value d_m of corresponds to the dynamical dimension for the mth segment. For a time quantum Δt, the particle of the first type, located on the segment m and, with the probability $\lambda_{1,m}\Delta$ move onto one cell in the direction of movement under the condition that the cell ahead is vacant. If a particle is in the cell (x,y), $x<K$ and the cell $(x+1,y)$, $y<Ml$ (the cell to the left of the particle) is vacant, the particle occupies the cell $(x+1,y)$ with the probability $\lambda_{2,m}\Delta$. If a particle of the second type is on the lane $x<K$, and the cell located to the left of this particle is occupied, then the particle does not move.

The paper [20] proposes an algorithm to estimate the probability that a particle transits to the utmost right lane successively and the distribution of flow intensities in a segment of segregation. In the model considered in [20] it is assumed that there are particles of two types moving a K-lane section divided into M segments. The length of the mth zone is equal to L_m cells.

Denote by $q_i(k,m)$ the intensity of the ith type particles movement along the kth lane in mth segment. In accordance with the intensity conservation law

$$q_1(k,m)=q_1(k,1),\ m=2,3,\ldots,M.$$

One may compute the intensities of the second type particles one may compute using the formulas

$$q_2(1,m)=q_2(1,m-1)(1-\beta(1,m-1)),$$

$$q_2(k,m)=q_2(k-1,m-1)\beta(k,m-1)+q_2(k,m)),$$

$$q_2(K,m)=q_2(K,m-1)\beta(K-1,m-1)+q_2(K,m-1),$$

where $\beta(K,m)$ is the probability that a particle of the second type starting to move along the kth lane in the mth segment, $k=1,\dots,K$, $m=1,\dots,M$, will come to the $(K+1)$ th lane before the end of the segment.

The value $\beta(k,m)$ is evaluated with formula

$$\beta(k,m)=0.5+\Phi\left(\frac{L_m-a(k,m)}{\sigma}\right),\ k=1,\dots,K,\ M-1,$$

where $\Phi(x)$ is the Laplace function.

$$\Phi(x)=\frac{1}{2\pi}\int_0^x e^{-z^2/2}dz,$$

$$a(k,m)=\frac{v_m\Delta t}{p(i,1)},$$

$$\sigma^2(k,m)=\frac{p(k,m)(v_m\Delta t)^2}{(1-p(k,m))^2},$$

$$a(k,m)=\frac{v_m\Delta t}{p(i,1)},$$

v_m is the deterministic-stochastic velocity for the mth segment.

The probability that a particle transits to the utmost right lane successively is

$$a=(q_2(K,M)+\beta(K-1,M-1)q_2(K-1,M))/\sigma_{k=1}^{K}q_2(k,1).$$

Calibration of the model parameters was carried on the base of data obtained as results of road measurement. The segregation section of the 4000 meter length was considered.

In the experiment, the section was divided into 40 segments of the length 100 meter.

The dependences of flow intensities for different lengths and segments are shown in Table 1.

The paper [18] proposes a stochastic model of flow segregation, Fig. 34 The traffic flow is represented a particle flow located on a two-lane segregation section. Any particle tends to be at prescribed lane before the end of the section to continue its movement along the lane. The problem is to estimate the minimum sufficient length of the section.

At any discrete moment $i\Delta$, each particle is at a cell and, in the interval of the duration Δt, the particle tends onto one cell. In the accordance with the deterministic-stochastic approach, the cellular field moves in the direction corresponding to the direction of the traffic flow. It is prescribed to be at the first (right) lane at the end of the section. The section is divided into two zones. They are the far zone and the near zone (relative to the end of the section).

Table 1

Dependence of intensity q_2 1/sec on the road lane and segment, $K=5$, $M=40$, $L_m=100$ m, $d_m=25$ m, $v_m=10$m/sec. [20]

Number of lane\ Segment number	1	2	3	4	5		7	8
1	0,060	0,046	0,035	0,027	0,021	0,016	0,012	0,009
2	0,060	0,060	0,057	0,052	0,046	0,040	0,034	0,029
3	0,060	0,060	0,060	0,059	0,058	0,055	0,051	0,047
4	0,060	0,060	0,060	0,060	0,061	0,061	0,060	0,060
5	0,060	0,074	0,088	0,101	0,115	0,128	0,142	0,155
Number of lane\ Segment number	9	10	11	12	13	14	15	16
1	0,007	0,005	0,004	0,003	0,002	0,002	0,001	0,001
2	0,024	0,020	0,017	0,014	0,011	0,009	0,007	0,006
3	0,043	0,039	0,034	0,030	0,026	0,023	0,019	0,016
4	0,058	0,056	0,054	0,051	0,048	0,045	0,042	0,038
5	0,167	0,179	0,191	0,202	0,212 0	0,222	0,231	0,239
Number of lane \Segment number	17	18	19	20	21	22	23	24
1	0,001	0,001	0,000	0,000	0,000	0,000	0,000	0,000
2	0,005	0,004	0,003	0,002	0,002	0,001	0,001	0,001
3	0,014	0,012	0,010	0,008	0,007	0,005	0,004	0,004
4	0,035	0,032	0,028	0,025	0,023	0,020	0,018	0,016
5	0,246	0,253	0,259	0,264	0,269	0,273	0,277	0,280
Number of lane\ Segment number	25	26	27	28	29	30	31	32
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,001	0,001	0,000	0,000	0,000	0,000	0,000	0,000
3	0,003	0,002	0,002	0,002	0,001	0,001	0,001	0,001
4	0,014	0,012	0,010	0,009	0,008	0,007	0,006	0,005
5	0,283	0,285	0,287	0,289	0,291	0,292	0,293	0,294
Number of lane\ Segment number	33	34	35	36	37	38	39	40
1	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
2	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
3	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000
4	0,004	0,003	0,003	0,003	0,002	0,002	0,002	0,001
5	0,295	0,296	0,297	0,297	0,298	0,298	0,298	0,299

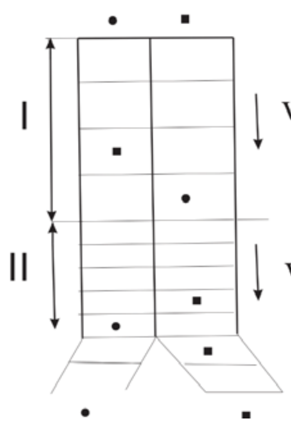

Fig. 4. Two-lane cellular field of segregation [18]

The rules of the movement of a particle in the mth zone, $m=1,2$, are the following. (1) If a particle is in «itself" lane, then this particle continues to be in this lane until the end of its movement. For the time interval of the duration Δt, with the probability $\lambda_{lo,m}\Delta + o(\Delta)$, a particle tends to move onto a cell forward if the cell ahead is vacant. (2) If a particle is not at "itself" lane, then, with the probability $\lambda_{isp,m}\Delta$, the particle comes to "itself" lane. (3) If a particle is at "itself" lane, the cell ahead, located at the adjacent lane, is occupied, then, with probability $\lambda_{i,sf,m}\Delta t$, then the particle moves forward. (4) If a particle is not in "itself" lane and the conditions (2), (3) are satisfied, the particle does not move. It is assumed $\lambda_{isf,2}=0$, i.e., the particle located in the second zone and is not in the "itself" zone, then the particle may not move forward without a transition to the "itself" lane.

The paper [18] proposes an approach to estimate the probability that a particle will come successively to the prescribed lane. In [18], it is assumed the following. The probability that in a cell, located in the mth lane and kth lane, there is a particle of the ith type $r_i(k,m)$ independently of the states of the other cells. The probability that this cell is vacant is $1-r(k,m)$,

$$r(k,m)=r_1(k,m)+r_2(k,m),\ i=1,2,\ k=1,2,\ m=1,2.$$

The probabilities and the intensities, $k=1,2$, satisfy the equations

$$q_i(i,2)=r_i(i,2)\,(v_2+\lambda_{lo,2}\Delta t(1-r(i,2))),\ i=1,2,$$

$$q_i(k,2)=r_i(k,2)\,v_2,\ i=1,2,\ k\neq i,$$

$$r(k,2)=r_1(k,2)+r_2(k,2)\leq 1,$$

$$r_1(k,2)>0,\ r_2(k,2)>0.$$

For any $k=1,2$, this system of equations is reduced to a quadratic equation. If there are more than one solutions of the system, then we choose the minimum values of $r_1(k,2)$ and $r_2(k,2)$.

Denote by $\eta(i,m)$ the increase of the non-moving coordinate system. The approach to evaluate the characteristics of $\eta(i,m)$ is developed. This approach is similar to the approach proposed in [20] and described above.

A numerical example is represented in [18]. It is assumed that

$$d(v)=a_0+a_1v+a_2v^2,$$
$$a_0=5.7m,\ a_1=0.5\ sec\ \ a_2=0.03\ sec^2,\ \Delta t=0.8\ sec,$$
$$\lambda_{sp,1}=\lambda_{sp,2}=11/sec$$
$$\lambda_{sf,1}=\lambda_{of,1}=\lambda_{sf,2}=\lambda_{0f,2}=0.5\ 1/sec,\ v_1=10m/sec$$
$$\rho_1(1,1)=\rho_2(1,1)=\rho_1(2,1)=\rho_2(2,1)=0.0025\ 1/m,$$
$$L_1=L_2=20m,\ v_1=3m/sec,\ v_2=5m/sec.$$

The dependences $r(2)$, $\beta(2)$ on v_2 are represented in Table 2.

Table 2

Dependencies $r(2)$ and $\beta(2)$ from v_2 (m/sec) [18]

v_2	3	5	7
$r(2)$	0.	0.422	0.365
$\beta(2)$	0.993	0.987	0.946

3. Modeling traffic flows on crossroads

The scheme of transport intersection planning and traffic management at an intersection must ensure to minimize the number of road accidents and time delays of vehicles [23], [24]. A decrease of the velocity of traffic at regulated intersections with a high density occurs due to the prolonged phase of the traffic light prohibitory signal and the organization of right turn traffic. At regulated crossroads, the intensity of vehicles turning to the right makes it difficult for the public transport to move along the designated line and causes a queue to form in the second lane. Vehicles turning from the second lane create a queue. This leads to that maneuver are impossible for several cycles. A congestion is formed at the crossroad. The percolation of vehicles into the designated lane is difficult because the quantity of vehicles is great. The queue of the public transport vehicles causes the increase of the temporary delays for these vehicles.

A scheme of the road intersection studied in [23], [24] and its geometrical dimensions are represented in Figure 5 and Table 3.

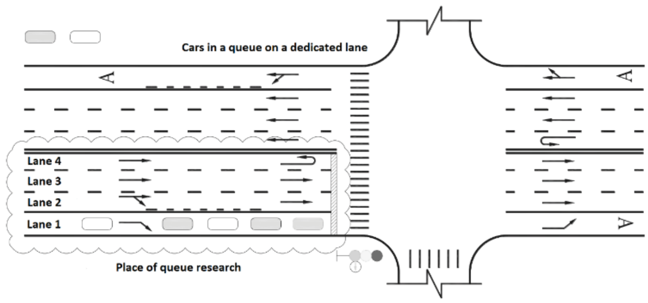

Fig. 5. Scheme of intersection

Table 3

Geometric dimension of intersection [24]

Parameter	Meaning
Number of lanes (main direction)	4
Width of the selected strip, m	4,0
Width of the carriageway lane, m	3,3
The duration of the traffic light cycle, s	150
Duration of the green signal (right turn), s	50
Duration of the green signal (moving straight), s	80

Measurements were caried out on the road network of Moscow to evaluate the delays of the vehicles at a regulated intersection. The papers [23], [24] consider a mathematical model of the crossroad that is located at the address: Leningradskiy prospect, 40.

There are four lanes on the main carriageway of this crossroad. These are the following lanes. The rightmost lane is a dedicated lane. Only public transport vehicles may be located at the lane. The vehicles located in this lane move forward and perform a right turn. In the second lane there is a forward movement. A small portion of the vehicles moving in the second lane slows down waiting for the possibility of changing lanes to the right to make a right turn. In the third lane vehicles move only forward. From the fourth lane, left-turn maneuvers are performed, and a small portion of the transport flow move in the forward direction.

The transport flow intensity in the fourth lane has no effect on the characteristics of traffic in the dedicated public traffic lane. Hence the main component of the transport flow is formed in the first, second and third lanes. Measurements of the traffic intensity on different weekdays and times of the day during the busiest days and hours to obtain the transport flow characteristics at different traffic intensities, with a dry surface.

Any vehicle passing through the crossroad is assigned to one of the following types.

The buses are assigned to the type 1, the trucks are referred to the type 2, and the cars are assigned to the type 3. In [24] the results of measurements are represented for the following four-time intervals: from 5 a.m. to 7 a.m., from 7 a.m. 9 a.m., from 2 p.m. to 4 p.m., from 7 p.m. to 9 p.m. The number of light phase cycles is more than 100 in each measurement. In accordance with the results of calculations, the length of the cars queue is nearly directly proportion to the traffic intensity. In the case of a small traffic load, the queue is also small or there is no queue.

The results of the measurements may be used as input data and settings for the parameters of the traffic mathematical model for a highway with various types of vehicles. If the traffic load is great, then, during a cycle, the queue decreases but does not disappear at all.

The dependence of the queue length on the load is represented in Figure 6.

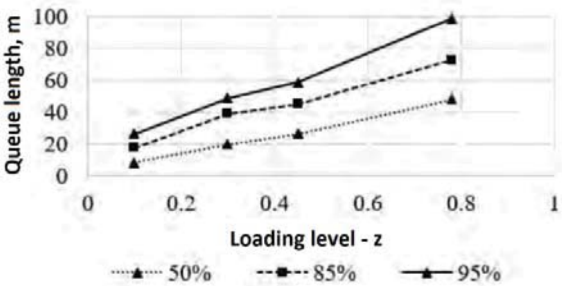

Fig. 6. Dependence of queue length on load

Papers [23], [24] consider a two-lane traffic model similar to the model is considered in [20]. A particle of the first type do not tend to change lanes. A particle of second type tends to transit to the utmost right lane. The section is divided into M segments. The length of each segment is equal to l cells. The dimension of any cell, located of the mth segment equals d_m, $m = 1, \ldots, M$.

If a particle of the first type is in the kth lane and the mth segment, then, during a time step, the particle tends to move with the probability $\lambda_{1,m}\Delta$. The attempt is realized if the cell ahead is vacant. If a particle of the first lane and the adjacent cell is occuoied, then the particle does not move.

Denote by $r_i(k,m)$ the there is a particle of the ith type in a cell of the kth lane and the mth zone.

An approach similar to the approach proposed in [20] is used to evaluate the flow characteristics.

A numerical example is considered in [24]. It is assumed that $M=5$, $\Delta = 0.9$ sec, $\lambda_{1,1} = \lambda_{1,2} = 0.5$ 1/sec, $\lambda_{2,1} = \lambda_{2,2} = 1$1sec, $r(1,1) = 0.56$ 1/m, $r(2,1) = 0.16$ 1/m, $L_m = 20$ m, $d_m = 20$ m.

In Table 4, the results of computations are represented.

Table 4

Dependences of α and r(2,M) on v_m

v_m	9	1	15
r(2,5)	0,348	0,331	0,318
α	0,999	0,988	0,949

Conclusion

The paper describes traffic mathematical models for traffic flows on highway segments located before road intersections. The traffic flow is represented in these models by particles moving on a cellular field. The models are based on the deterministic-stochastic approach to traffic modeling. The calibration of the models was fulfilled on the base of measurements on highways of Moscow. The models may be used for optimization of traffic control. These problems are also related to problems of the ecology and the transport safety.

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МАТЕМАТИЧЕСКИЕ МОДЕЛИ АВТОМОБИЛЬНЫХ ПОТОКОВ НА АВТОМОБИЛЬНЫХ ДОРОГАХ С ПЕРЕСЕЧЕНИЯМИ И СОЕДИНЕНИЯМИ

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Аннотация

Рассмотрены математические модели движения автотранспорта на участках автомагистралей вблизи перекрестков или участков сегрегации потоков. В этих моделях частицы, соответствующие автомобилям, движутся по вероятностным правилам вдоль клеточного поля, которое движется с постоянной скоростью в направлении, совпадающем с направлением движения частиц. Поле ячеек состоит из последовательностей ячеек. Каждая такая последовательность соответствует полосе на шоссе. Масштаб времени в модели дискретный или непрерывный. Модель представляет собой динамическую систему с дискретным пространством состояний и дискретным или непрерывным временем. Математическое описание модели также может быть представлено в терминах клеточного автомата или случайного процесса с запретами. В любой момент времени в каждой ячейке находится не более одной частицы. При каждом движении частица либо перемещает одну клетку в направлении движения, либо перемещается на следующую полосу движения, либо остается на месте. Скорость транспортного потока на участке шоссе соответствует сумме заданной скорости поля ячейки и средней скорости частиц относительно поля. Изучаемые характеристики – это скорость транспортного потока, его интенсивность и вероятность успешной перестройки транспортного средства на рассматриваемом участке трассы. При настройке параметров модели используются данные измерений характеристик транспортных потоков на автомагистралях. Разработаны аналитические подходы к оценке изучаемых характеристик. Созданы компьютерные программы для реализации разработанных алгоритмов расчета. Приведены результаты расчетов.

Ключевые слова: *математические модели движения, характеристики автомобильного движения, многополосное движение.*

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