APPROXIMATION OF THE AVERAGE WAITING TIME FOR THE HE2/HE2/1 QUEUING SYSTEM USING SIMULATION

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Kada,Othmane,
PSUTI, Samara, Russia, ***otman2333@mail.ru***

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In this article we present a simulation based method to approximate the average waiting time for the queuing system (QS) HE2/HE2/1 , Which is by Kendall's definition belonging to the class G/G/1, with probabilistic Mixing Distribution of second order hypererlangian distribution inputs for both inter arrival and service time functions Our methods consists of creating a virtual model of Traffic flows ,first by using different methods and algorithms of generating random numbers from hypererlangian distribution using a random variate generator and a Discrete event simulator based on a queuing system (QS) of type HE2/HE2/1 then use the results to analyse the behavior of the system during different etapes of execution , The results obtained for the average waiting time from our simulation method are very identical to our theoretical results ,in the end this work leads us to evaluate accuracy of our theoretical methods. and to collect a big data-set that can be used for other proprieties to find a solution for real problems of modern teletraffic theory.

Information about author:
Kada,Othmane, *PhD student, PSUTI, Samara, Russia*

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I. Introduction

Traffic flows analysis is a very important part of studying the performance of modern telecommunications network because it gives a better understanding about the resource usage and the average waiting time inside the system which we will study in our work queuing theory techniques involves the mathematical study of traffic flows behavior by modeling it according to the main characteristic of queuing systems which can be described according to Kendall's notation in a basic form by A/B/m, where *A* indicates the distribution of inter-arrivals, *B* denotes the distribution of service duration, *m* is the number of servers.

Usually modelisation of traffic flows in modern telecommunications network the G/G/1 queuing system is often used where the number of servers is one and a general form of distribution law functions is used, such as erlangian, exponential, hyperexponential, hyperelangian, etc., for which the variation coefficients of random variables are greater than or equal to 1 ($c_\tau \geq 1$) or less than 1 ($c_\tau < 1$).

The coefficient of variation greater than 1 indicates that the probability of the appearance of large values of a random variable is much higher than that of the classical exponential distribution, and the "tail" of the distribution is more powerful. For the coefficient of variation of a smaller one is the opposite, Such system is a typical problem of a discrete event system, and the computer simulation is a quite effective way for analyzing the performances by calculating the average waiting time which From the queuing theory [1-4] in a G/G/1 queue is determined by

$$\overline{W} = \frac{D_\lambda + D_\mu + (1-\rho)^2/\lambda^2}{2(1-\rho)/\lambda} - \frac{\overline{I^2}}{2\overline{I}}, \quad (1)$$

where ρ is the system load factor ($0 < \rho = \lambda/\mu < 1$), λ is the intensity of the input stream, μ is the service intensity, D_λ, D_μ respectively the variance of the arrival intervals and the service time, $\overline{I}$, $\overline{I^2}$ respectively the average value and the second initial moment of the idle period in this paper we will present a different way to approximate the average waiting time in the queuing system by creating a virtual traffic flow using a Discrete Event Simulator.

II. Stateme of the problem

Our task is to find the average waiting time of the HE2 / HE2 / 1, for this the classical method of spectral decomposition of the solution of the Lindley integral equation is widely used but for our case, the problem is that when studying such systems as HE2/HE2/1 (QS) is difficult to find a solution for the average waiting time due to high computational complexity, that is why we will use a simulation based solution to approximate the average waiting time of this queuing system.

III. solution of the problem

We take the HE_2 / HE_2 / 1 (QS) with probabilistic density functions (pdf) in the inputs.

inter-arrivals function:

$$a(t) = 4p\lambda_1^2 te^{-2\lambda_1 t} + 4(1-p)\lambda_2^2 te^{-2\lambda_2 t}, \quad (2)$$

service time function:

$$b(t) = 4p\mu_1^2 te^{-2\mu_1 t} + 4(1-p)\mu_2^2 te^{-2\mu_2 t}. \quad (3)$$

Inverse-transform Technique

The inverse-transform technique can be used in principle for any distribution having density functions (pdf) f(x). And Most useful when the Cumulative distribution function CDF F(x) has an inverse F-1(x) which is easy to compute. It consists of 4 steps example on Exponential distribution.

Take the exponential distribution function with the PDF $f(X) = \lambda e^{-x\lambda}$.

1. find the CDF F(X) by integrate the PDF f(X), $F(X) = \int_{-\inf}^{X} f(X)$:

$$F(X) = \int_{-\inf}^{X} f(X) = 1 - e^{-x\lambda}$$

2. Set F(X)= R on the range of X: $1 - e^{-x\lambda} = R$.
3. Solve the equation F(X)= R for X in terms of R:

$$X = \frac{-1}{\lambda}\ln(1-R).$$

4. nerate uniform random numbers R_1, R_i and compute the random variate by $X_i = F^{-1}(R_i)$ where R_i is a uniformly distributed random number on (0,1) exponential variate can be generated by

$$X_i = \frac{-\ln(1-R_i)}{\lambda}.$$

Example of anexponential variate generator algorithm.

```
Function Rexp(λ):
        R = random()
        return(-ln(1-R)/ λ)
```

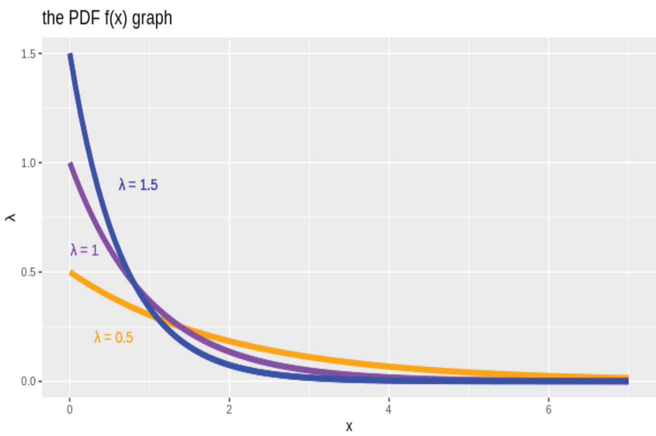

Fig. 1. Exponential PDF graphs

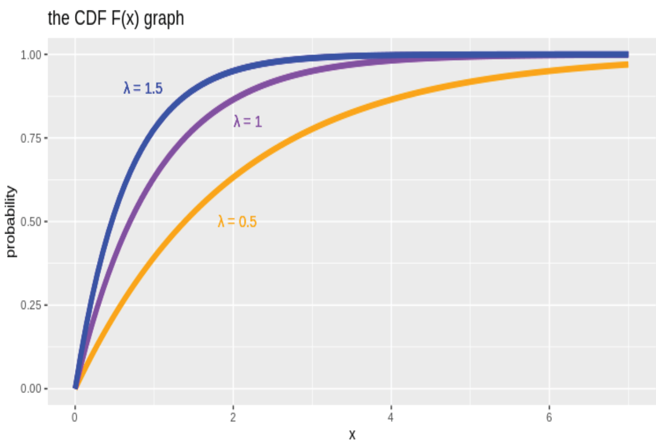

Fig. 2. Exponential CDF graphs

Convolution Generation Technique

The distribution of the sum of two or more random variables is called the convolution. Let Y_i<G(y) be IID random variables $Y=\sum_{i=1}^{n} X_i$. Then the distribution of Y is said to be the convolution of X_i.

Example of erlang distribution:

take Y_1 and Y_2 two exponential distribution function with the rate λ=2λ where:

$(X_1)=2\lambda e^{-x(2\lambda)}$,$(X_2)=2\lambda e^{-x(2\lambda)}$

$Y=(X_1)+(X_2)=2\lambda e^{-x(2\lambda)}+2\lambda e^{-x(2\lambda)}$

lets find the CDF of X

$CDF(Y)=P(X_1+X_2\ \ x)$

$CDF(Y)=\int_{x_1} P(X_1+X_2\ \ x\vee X_1).P(X_1)dx_1$

$CDF(Y)=\int_0^x P(X_2\ \ x-X_1).2\lambda e^{-x_1.(2\lambda)}dx_1$

$$CDF(Y)=\int_0^x P(X_2\ \ x-X_1).2\lambda e^{-x_1.(2\lambda)}dx_1$$

$CDF(Y)=\int_0^x \left(1-e^{-2\lambda(x-X_1)}\right)2\lambda e^{-2\lambda X_1}dx_1$

$CDF(Y)=1-e^{-2\lambda x}-2\lambda xe^{-2\lambda x}$

Then finding the PDF from the CDF:

$PDF(Y)=\frac{dx}{x}PDF(Y)=\frac{dx}{x}\left(1-e^{-2\lambda x}-2\lambda xe^{-2\lambda x}\right)$

$PDF(Y)=4\lambda^2xe^{-2\lambda x}$.

Got the distribution function with the PDF $f(X)=4\lambda^2xe^{-2\lambda x}$ and CDF $F(x)=1-e^{-2\lambda x}-2\lambda xe^{-2\lambda x}$which is an Erlang distribution with shape 2 and rate =2λ, that means that if Y is Erlang distributed with parameters (shape=K) and (rate= λ), then Y can be expressed as a sum of independent exponential X_i with the shape of K, so Erlang variate can be generated with $Y_i=\sum_{1=1}^{K}\frac{-ln(1-R_i)}{\lambda K}$ in the case of *k*=2 and rate = 2λ, $Y_i=\sum_{1=1}^{2}\frac{-ln(1-R_i)}{4\lambda}$.

Example of an Erlang variate generator algorithm.

```
Function Rerlang (λ, k):
    s=0
    for (i from 1 to k ) do:
            s=s+Rexp(λ*k)
    return( S )
```

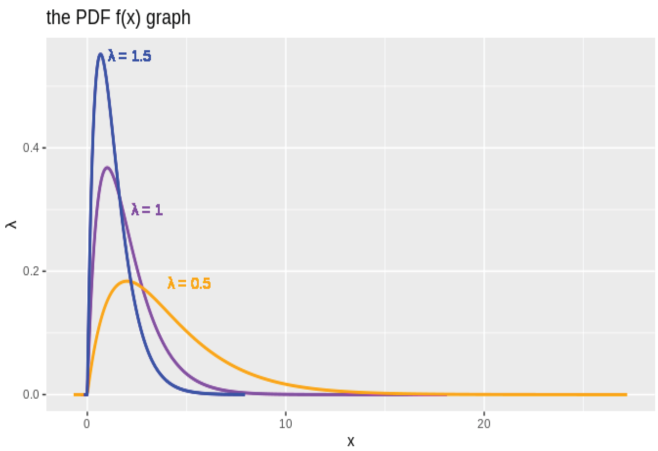

Fig. 3. Erlang PDF graphs

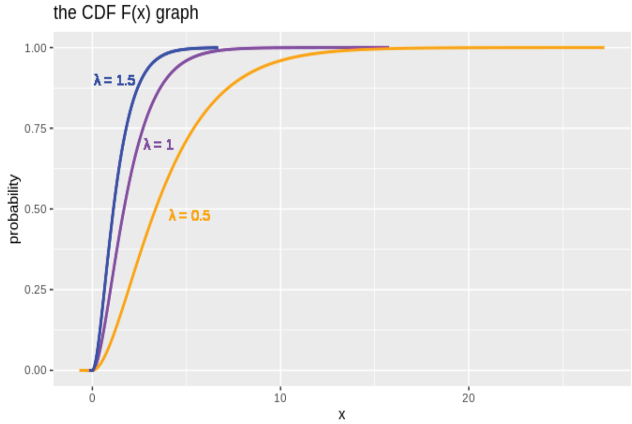

Fig. 4. Erlang CDF graphs

Composition Technique

This method applies when the distribution function F can be expressed as a mixture of other distribution functions F_1,F_2, F_i

$F_i=\sum_{1=1}^{K} p_iF_i$ where $p_i\geq 0,\sum_{1=1}^{\infty} p_i=1$.

This method is useful if it is easier to sample from the F_i than from F.

First generate an index I such that P(I=i)=p_i, i=1,2. Then Generate a random variable X with distribution function F_I.

Example of hyper erlang distribution.

Take HE_2 the probabilistic mixture of two erlang distribution of second order with rate=2λ

$E_2(x)=4\lambda^2xe^{-2\lambda x}$ $HE_2=\sum_{1=1}^{2} p_iE_2$

where $p_i\geq 0,p_1=p,p_2=(1-p)$

$HE_2(x)=4p\lambda_1^2xe^{-2\lambda_1 x}+4(1-p)\lambda_2^2xe^{-2\lambda_2 x}$

$CDF(HE_2)=p\left(1-e^{-2\lambda_1 x}-2\lambda_1xe^{-2\lambda_1 x}\right)+$

$+(1-p)\left(1-e^{-2\lambda_2 x}-2\lambda_2xe^{-2\lambda_2 x}\right)$.

Hyper Erlang with shapes (2,2) and rates ($2\lambda_1$,$2\lambda_2$) and probabilistic mixtures (p,1-p) variate with can be generated with

$HE_i=p\sum_{1=1}^{2}\frac{-ln(1-R_i)}{4\lambda_1}+(1-p)\sum_{1=1}^{2}\frac{-ln(1-R_i)}{4\lambda_2}$.

Example of an hyper Erlang variate generator algorithm.

```
Function RhyperErlang (Vλ ,Vp, Vk):
    s=0
    for (i from 1 to  size (Vp) ) do:
            for (j from 1 to  size (Vλ) ) do:
                    s=s+Rerlang (Vλj,Vki)
            s=s*Vpi
    return( S )
```

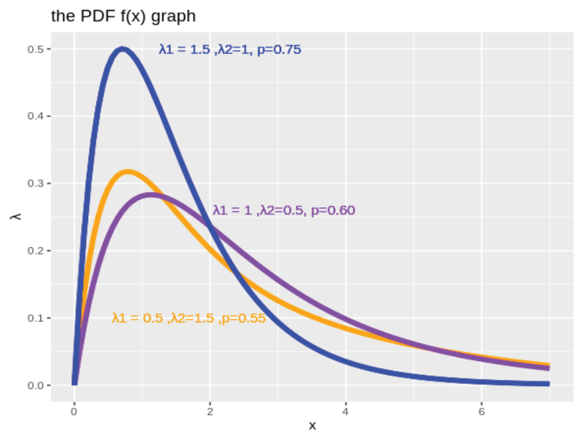

Fig. 5. Hyper Erlang PDF graphs

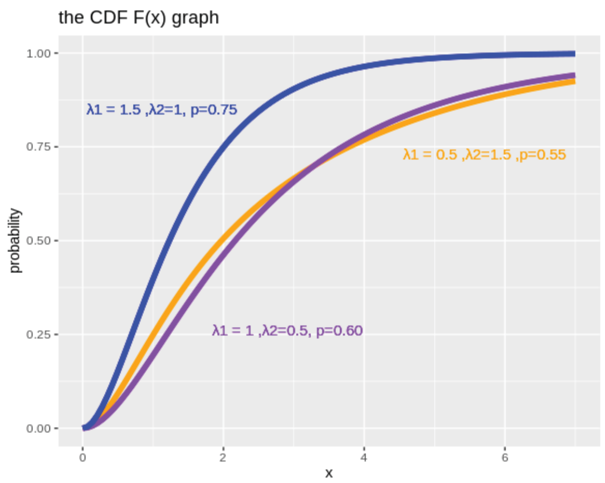

Fig. 6. Hyper Erlang CDF graphs

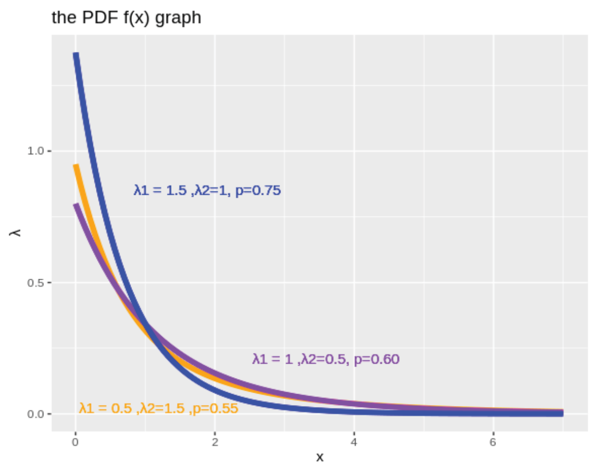

Fig. 7. Hyper Exponential PDF graphs

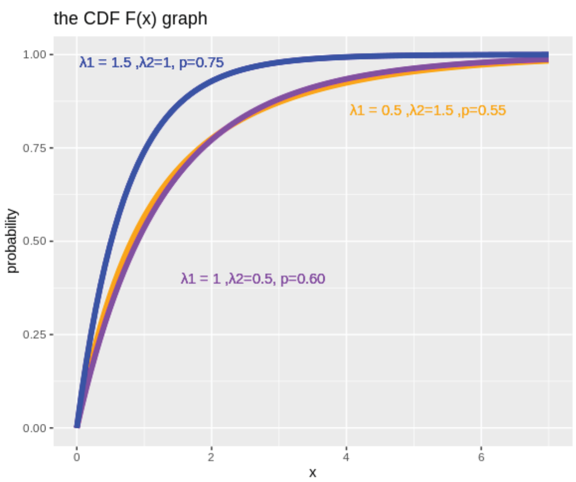

Fig. 8. Hyper Exponential CDF graphs

Example of hyper erlang distribution.

Take H_2 the probabilistic mixture of two exponential distribution of second order with rate=λ

$$H(x) = \lambda x e^{-\lambda x}$$

$$H_2 = \sum_{1=1}^{2} \; p_i H \text{ where } p_i \geq 0, p_1 = p, p_2 = (1-p)$$

$$H_2(x) = p\lambda_1 x e^{-\lambda_1 x} + (1-p)\lambda_2 x e^{-\lambda_2 x}$$

$$CDF(H_2) = p\left(1 - e^{-\lambda_1 x}\right) + (1-p)\left(1 - e^{-\lambda_2 x}\right)$$

Hyper exponential with rates (λ_1, λ_2) and probabilistic mixtures (p,1-p) variate can be generated with

$$H_i = p\frac{-ln(1-R_i)}{\lambda_1} + (1-p)\frac{-ln(1-R_i)}{\lambda_2}.$$

Example of an hyper Erlang variate generator algorithm.

```
Function RhyperExp (Vλ ,Vp):
    s=0
    for (i from 1 to size (Vp) ) do:
        for (j from 1 to  size (Vλ) ) do:
            s=s+Rexp (Vλj)
        s=s*Vpi
    return( S )
```

Discrete event simulator

Discrete Event Simulation (DES) is the simulation that modelate a complex system behavior an ordered sequence of discrete events collection, when each event have a known effect on the rest of the system state at a specific point in time.

Each process inside the system can defined based on their impact on the system, their resource requirements, and their trigger way, may be scheduled or occurred randomly, or occurred responsively to other event in the system, in the end they all will be combined within the simulation to modelate the system from the scratch application on queuing system:

In the queuing system model two types of events are used, arrival and departure. The arrival is when a customer reaches a service station, and the departure corresponds to the event when the customer leaves the system. And logically the arrival event for a customer is executed before its departure event example of $HE_2/HE_2/1$ queue simulation algorithm:

```
function queue (arrivals,servicing,n,k)
    q=[]
    for i (from1 to n) do:
        ai=RhyperErlang(arrivals)
        bi=RhyperErlang(servicing)

    a=cumsum(ai)
    k=1
    q=append(q,steps(a,b,k,n))

function steps (a,b,k,n)
    Sort (a,b) in terms of a(ascending)
    Create vector pn
    Create vector sR
    Create vector dn
    sk=0
        for i (from1 to n) do
            p=arg min(s)
            spi=max(ai,spi) +bi
            di=spi
        end for
    Put (a,d,p) back to original (input) ordering of a
    return(d,p)
```

Results

We made our simulation based R language using RStudio and a list of useful l libraries .Distr :an Object Oriented Implementation of Distributions, *Simmer*: a process-oriented and trajectory-based Discrete-Event Simulation, Parallel: Support for Parallel computation.

To start simulation we have as an inputs inter-arrivals function (1) with the variables (λ_1,λ_2,p) and the servicing function (2) with the variables (μ_1,μ_2,q),choosing the best combination value of the variables to get the best simulation experience is very important that why we used the combination from our theoretical solution to the approximation to the average waiting time to the queuing system $HE_2/HE_2/1$ [2], where we have:

$$\lambda_1 = 2p/\bar{\tau}_\lambda, \quad p = \frac{1}{2} \pm \sqrt{\frac{2(1+c_\lambda^2)-3}{8(1+c_\lambda^2)}}$$

$$\mu_1 = 2q/\bar{\tau}_\mu, \quad \mu_2 = 2(1-q)/\bar{\tau}_\mu, \quad q = \frac{1}{2}\left(1 \pm \sqrt{\frac{c_\mu^2-1}{c_\mu^2+1}}\right)$$

$$\rho = \bar{\tau}_\mu / \bar{\tau}_\lambda, \quad \bar{\tau}_\mu = 1$$

where ρ refers to the server utilization, take

.

Table 1

Inputs combination for a(t) and b(t)

ρ	(c_λ,c_μ)	(λ_1,λ_2,p)	(μ_1,μ_2,q)
0,1	(0,71;0,71)	(0.1052210, 0.094779001, 0.5261050)	(1.052210, 0.94779001, 0.5261050)
0,5	(0,71;0,71)	(0.5261050, 0.473895007, 0.5261050)	(1.052210, 0.94779001, 0.5261050)
0.9	(0,71;0,71)	(0.9469890, 0.853011013, 0.5261050)	(1.052210, 0.94779001, 0.5261050)

Replacing (λ_1,λ_2,p) in a(t) (1) and (μ_1,μ_2,q) in b(t) (2) we get:

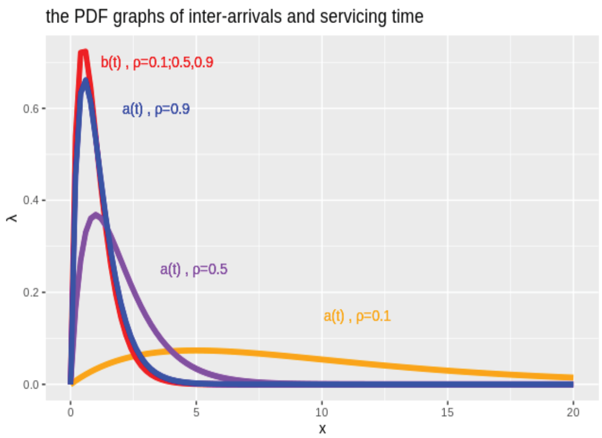

Fig. 9. PDF graphs for a(t) and b(t)

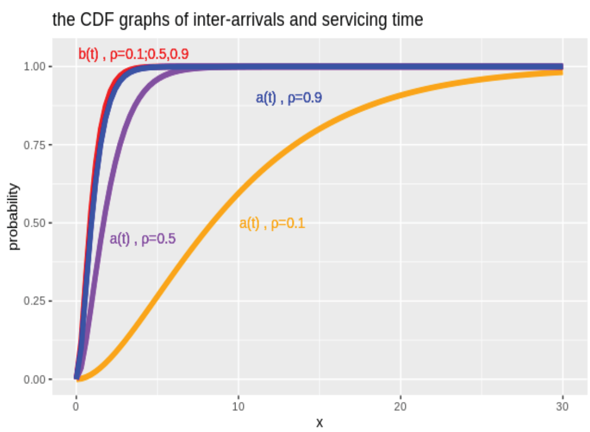

Fig. 10. CDF graphs for a(t) and b(t)

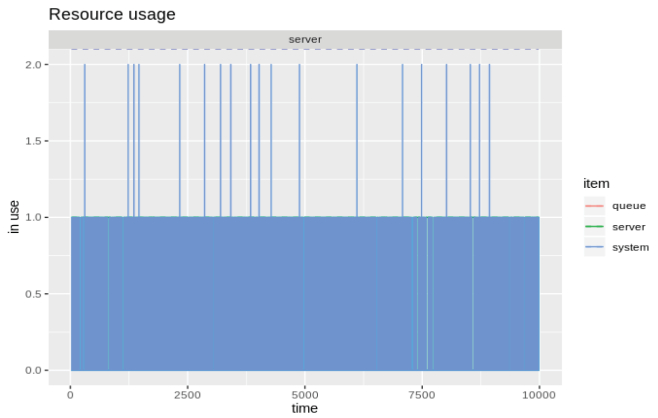

Fig. 11. Resource Usage by step, ρ=0.1

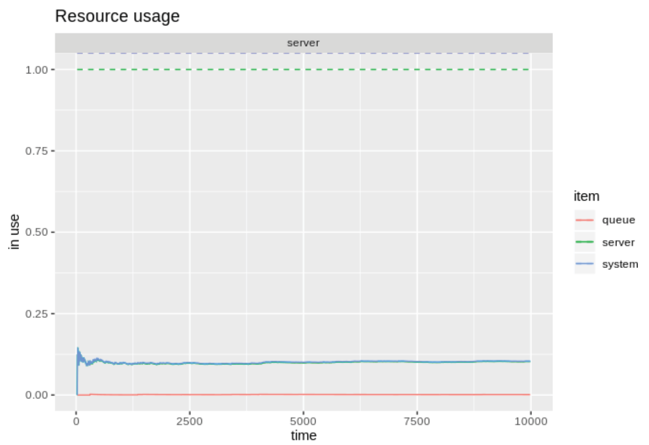

Fig. 12. Resource Usage, ρ=0.1

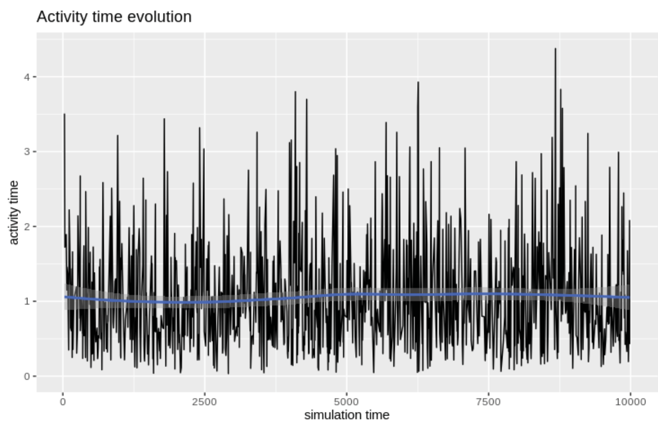

Fig. 13. Servicing time, ρ=0.1

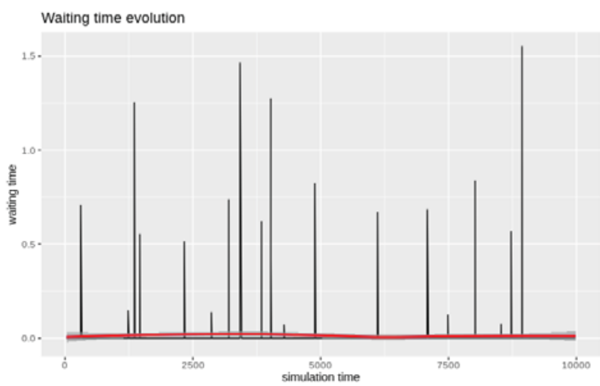

Fig. 14. Waiting time, ρ=0.1

	name	start_time	end_time	activity_time	finished	replication	waiting_time
1	arrival0	24.91055	28.41832	3.50776938	TRUE	1	8.881784e-16
2	arrival1	34.44206	36.16511	1.72305177	TRUE	1	0.000000e+00
3	arrival2	56.87647	58.76803	1.89155824	TRUE	1	1.776357e-15
4	arrival3	69.08397	70.23390	1.14992583	TRUE	1	4.440892e-16
5	arrival4	71.83395	73.29041	1.45645337	TRUE	1	0.000000e+00
6	arrival5	84.85442	86.21450	1.36007235	TRUE	1	5.107026e-15
7	arrival6	101.94744	102.29720	0.34976099	TRUE	1	3.497203e-15
8	arrival7	109.66216	111.88341	2.22125130	TRUE	1	1.332268e-15
9	arrival8	132.87503	134.05040	1.17537088	TRUE	1	9.992007e-15
10	arrival9	135.14059	135.78928	0.64869888	TRUE	1	9.325873e-15
11	arrival10	148.38461	148.99106	0.60644145	TRUE	1	5.218048e-15

Fig. 15. Head of queuing results table, ρ=0.1

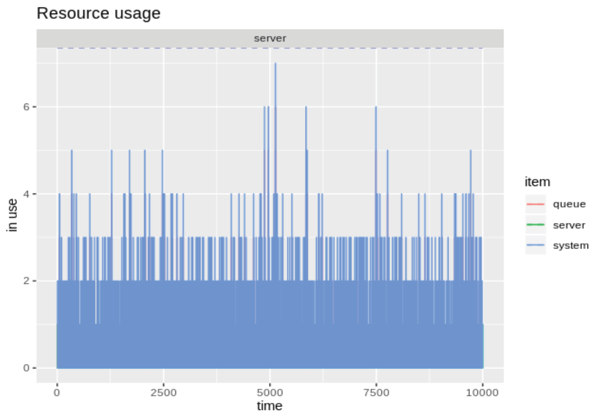

Fig. 16. Resource Usage by step, ρ=0.5

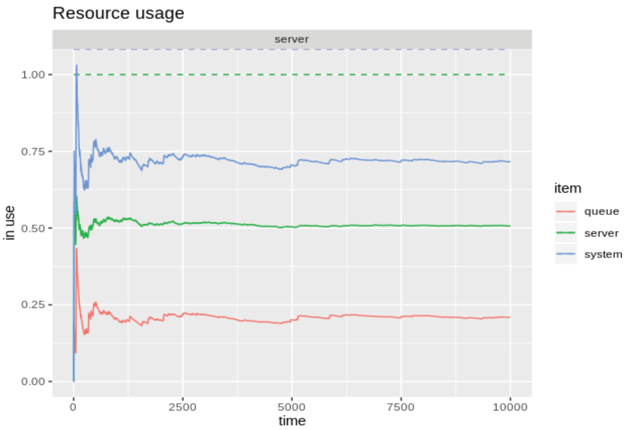

Fig. 17. Resource Usage, ρ=0.5

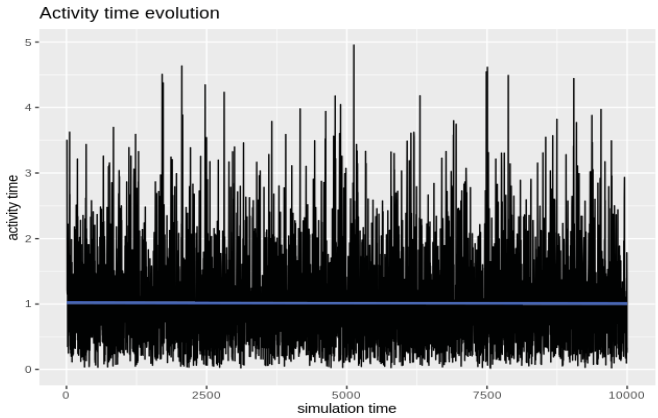

Fig. 18. Servicing time, ρ=0.5

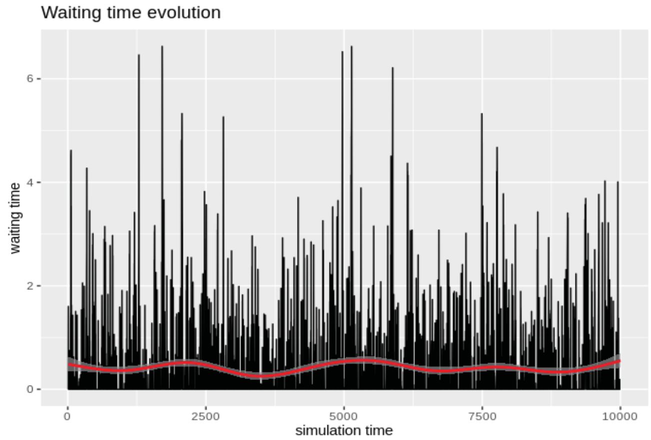

Fig. 19. Waiting time, ρ=0.5

	name	start_time	end_time	activity_time	finished	replication	waiting_time
1	arrival0	4.982109	8.489879	3.50776938	TRUE	1	8.881784e-16
2	arrival1	6.888412	10.733320	2.24344149	TRUE	1	1.601467e+00
3	arrival2	10.334515	11.954070	1.22074993	TRUE	1	3.988046e-01
4	arrival3	14.117632	15.267558	1.14992583	TRUE	1	4.440892e-16
5	arrival4	14.667628	16.569605	1.30204708	TRUE	1	5.999297e-01
6	arrival5	17.580535	18.940607	1.36007235	TRUE	1	1.554312e-15
7	arrival6	20.999138	21.348899	0.34976099	TRUE	1	0.000000e+00
8	arrival7	22.542082	24.763333	2.22125130	TRUE	1	1.332268e-15
9	arrival8	27.184657	28.360027	1.17537088	TRUE	1	0.000000e+00
10	arrival9	27.637767	29.684430	1.32440282	TRUE	1	7.222605e-01

Fig. 20. Head of queuing results table, ρ=0.5

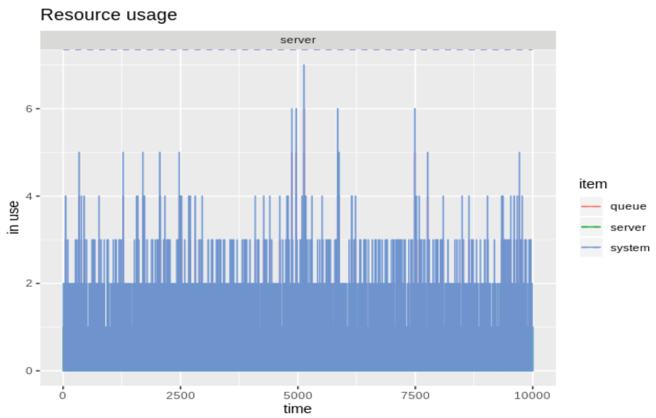

Fig. 21. Resource Usage by step, ρ=0.9

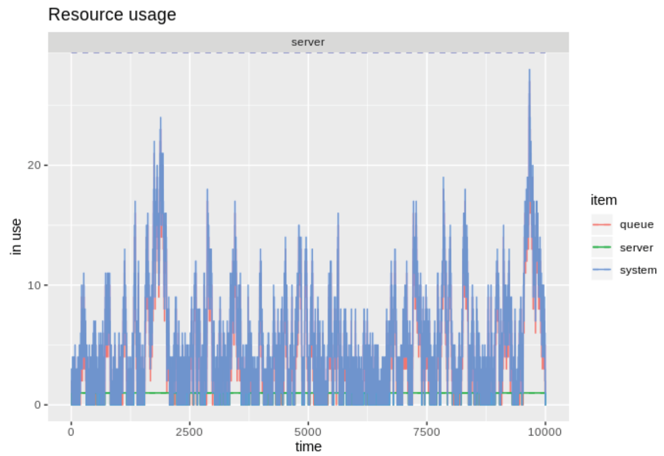

Fig. 22. Resource Usage, ρ=0.9

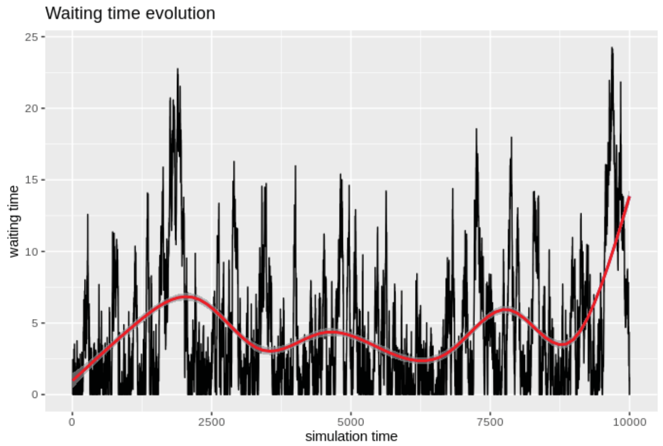

Fig. 23. Servicing time, ρ=0.9

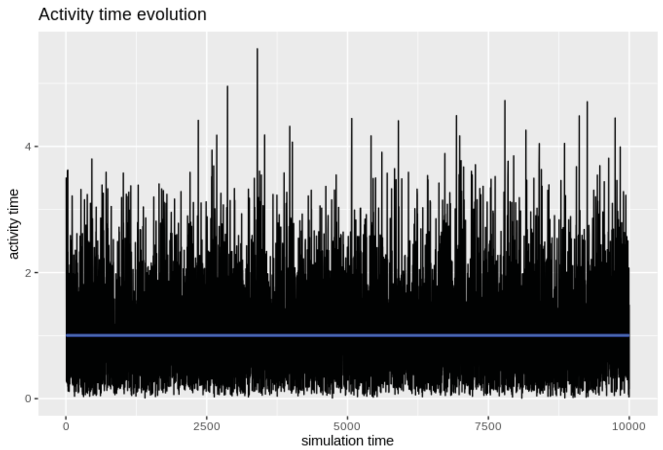

Fig. 24. Waiting time, ρ=0.9

	name	start_time	end_time	activity_time	finished	replication	waiting_time
1	arrival0	2.767838	6.275608	3.50776938	TRUE	1	4.440892e-16
2	arrival1	3.826896	8.167166	1.89155824	TRUE	1	2.448712e+00
3	arrival2	5.741397	9.387916	1.22074993	TRUE	1	2.425769e+00
4	arrival3	8.234110	9.662914	0.27499805	TRUE	1	1.153806e+00
5	arrival4	9.511806	10.964961	1.30204708	TRUE	1	1.511084e-01
6	arrival5	11.130087	12.490159	1.36007235	TRUE	1	0.000000e+00
7	arrival6	13.029311	13.379072	0.34976099	TRUE	1	0.000000e+00
8	arrival7	13.886502	16.107753	2.22125130	TRUE	1	0.000000e+00
9	arrival8	16.465710	17.641081	1.17537088	TRUE	1	0.000000e+00
10	arrival9	16.717438	18.247523	0.60644145	TRUE	1	9.236429e-01
11	arrival10	17.438215	19.199754	0.95223093	TRUE	1	8.093078e-01
12	arrival11	18.909774	19.738379	0.53862530	TRUE	1	2.899801e-01

Fig. 25. Head of queuing results table, ρ=0.9

Table 2 show the comparison between simulation and theoretical results of the average waiting time for the $H_2/H_2/1$ and $HE_2/HE_2/1$ systems for the cases of low, medium and high loads $\rho = 0,1; 0,5; 0,9$.

Here we take into account that the variation range of variation coefficients of the arrivals intervals of input flow c_λ and service time c_μ for the distribution of HE_2 is wider than that of the distribution of H_2. Taking into account that the $H_2/H_2/1$ system is not applicable in cases $c_\lambda < 1$ and $c_\mu < 1$ to evaluate our simulation results we made a comparison with our theoretical results using mean absolute percentage error (MAPE) for calculate the loss and the Coefficient of Determination R-Squared for the score.

Table 2

Inputs parameters		Average Waiting Time for QS $H_2/H_2/1$		Average Waiting Time for QS $HE_2/HE_2/1$	
ρ	(c_λ, c_μ)	theoretical method	simulation method	theoretical method	simulation method
0,1	(0,71; 0,71)	-		0,02	0.02
	(2,2)	0,445	0.456	0,34	0.33
	(4,4)	1,779	1.912	1,68	1.81
	(8,8)	7,112	7.749	7,16	8.13
0,5	(0,71; 0,71)	-		0,401	0.40
	(2,2)	4,044	4.112	3,98	4.01
	(4,4)	16,129	16.622	16,53	16.63
	(8,8)	64,178	74.01	66,73	65.30
0,9	(0,71; 0,71)	-		4,299	4.35
	(2,2)	36,20	36.465	36,21	35.90
	(4,4)	144,833	150.330	145,31	135.10
	(8,8)	577,861	655.579	580,56	570.20

Table 3

Evaluation of the result from simulation results

Inputs parameters	Average Waiting Time for QS $H_2/H_2/1$		Average Waiting Time for QS $HE_2/HE_2/1$	
ρ	MAPE	R2Score	MAPE	R2Score
0,1	0.063	0.983	0.0606	0.971
0,5	0.0669	0.9521	0.0094	0.9993
0,9	0.0599	0.963	0.0271	0.999
All	0.0	0.9779	0.0324	0.9993

The results of Evaluation experiments demonstrate that the compared results between simulation and theoretical results are very similar where the Score ~= 97% with MAPE~=6% for QS $H_2/H_2/1$ and Score ~= 99% with MAPE~=3% for QS $HE_2/HE_2/1$.

Conclusion

The goal of this study was to find an approximation for the average waiting time of the queuing system of type $HE_2/HE_2/1$ using a simulation model Based on Lindley's definition.

The results obtained for the average waiting time are very similar to our theoretical results, from here we can confirm that our results for the approximation of the Average Waiting Time from both the theoretical and simulation results is correct and our proposed simulation model can accurately estimate the other main parameters of the queuing system. The advantage of using simulation is the generating of unlimited data-sets in form of traffic logs, and gives the ability to explore all the states of the queuing system which allows for other studies.

Another advantage is that analyzing the results leads to the detection of possible problems in the system and produce graphical representations which are very handy in the discussion of system refactoring and possible corrections and optimizations.

In the end we say that the obtained results can be used in the optimization of the modern teletraffic systems

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ОПРЕДЕЛЕНИЕ ПРИБЛИЖЕННОГО СРЕДНЕГО ВРЕМЕНИ ОЖИДАНИЯ ДЛЯ СИСТЕМЫ МАССОВОГО ОБСЛУЖИВАНИЯ HE2/HE2/1 С ИСПОЛЬЗОВАНИЕМ МОДЕЛИРОВАНИЯ

Када Отхмане, *ПГУТИ, Самара, Россия,* ***otman2333@mail.ru***

Аннотация

Представлен симуляционный метод, базирующийся на аппроксимации среднего времени ожидания в системах массового обслуживания, которую Kendall определяет принадлежащей к классам G/G/1, со смешанным вероятностным распределением второй очереди hypererlangian распределения для обеих интервальных и сервисных временных функций. Данный метод состоит в создании виртуальной модели загруженности, в первую очередь используя разнообразные методы и алгоритмы для генерации случайных чисел из (hypererlangian) распределения гиперэрланга используя генератор случайных чисел и дискретный симулятор событий, основанный на Системе Очередей (queuing system (QS)) типов HE2/HE2/1, при использовании полученных результатов для анализа поведения системы на различных этапах выполнения. Результаты, полученные для среднего времени ожидания от предложенного метода моделирования, подтверждают теоретические результаты. Таким образом, эта работа позволяет оценить точность теоретических методов, а также собрать большой набор данных, который может быть использован для поиска решения реальных проблем теории телетрафика.

Ключевые слова: *дискретный симулятор событий, Система массового обслуживания, распределение гиперэрланга, среднее время ожидания.*

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Информация об авторе:

Када Отхмане, *аспирант, ПГУТИ, Самара, Россия*