CAR WHEEL MODELS USING THE DISTRIBUTIONS LAWS OF FORCES ON A CONTACT PATCH WITH THE ROAD SURFACE

DOI: 10.36724/2072-8735-2021-15-12-72-80

Maria Yu. Karelina,
Moscow Automobile and Road Construction State Technical University, Moscow, Russia, **karelina@madi.ru**

Alexey V. Terentyev,
Moscow Automobile and Road Construction State Technical University, Moscow, Russia

Gregory M. Rosenblat,
Moscow Automobile and Road Construction State Technical University, Moscow, Russia, **gr51@mail.ru**

Vladimir B. Yashin,
Moscow Automobile and Road Construction State Technical University, Moscow, Russia, **hekkoki@gmail.com**

Manuscript received 26 November 2021;
Accepted 16 December 2021

Keywords: *Tire model, brush model, dry friction, contact patch*

It is natural to consider the rolling dynamics of an automobile wheel when it interacts with the road surface. At the same time the most difficult and important task is to determine the force components applied to the wheel, such as the friction driving force, the drag force, the rolling and spinning resistance moments that occur in the contact spot from the side of the roadbed. The paper investigates the aspects of dry friction, rolling and sliding of an automobile wheel presented as a deformable body. In this case, it is of great importance to take into account the treads, which is reflected in the tire models. An important aspect is the study of the laws of distribution of normal stresses in the contact area. To solve practical issues of road transport, approaches based on the Magic Formula of Pacejka and calculation methods based on brush, ribbon and string models, in particular, the Brush model of Svendenius, are highlighted. The conditions of its applicability are obtained and justified.

Для цитирования:
Карелина М.Ю., Терентьев А.В., Розенблат Г.М., Яшин В.Б. Модели автомобильного колеса, использующие законы распределения силовых воздействий в области контакта с дорожным покрытием // T-Comm: Телекоммуникации и транспорт. 2021. Том 15. №12. С. 72-80.

For citation:
Karelina M.Yu., Terentyev A.V., Rosenblat G.M., Yashin V.B. (2021). Car wheel models using the distributions laws of forces on a contact patch with the road surface. *T-Comm*, vol. 15, no.12, pp. 72-80. (*in Russian*)

Introduction

When modeling the rolling dynamics of a wheel, the most difficult and important task is to determine the force components applied to the wheel, such as: the driving friction force, the resistance force, the rolling and spinning resistance moments that occur in the contact spot from the side of the roadbed (for an automobile wheel) or from the side of the rail (if we are talking about a railway wheel). For example, it is known that the friction force (clutch) applied to the driving wheel of the vehicle (a moment from the engine is applied to it) is always directed forward in the direction of movement, and a similar force applied to the driven wheel is directed (as a rule) backwards and slows down the movement of the vehicle.

It is clear that the model of the point contact of the wheel with the road is overly simplified and does not correspond to the real practical tasks of studying the dynamics of motor vehicles. With stationary and rectilinear motion, such a model is quite acceptable. The first results in this direction in 1779 were obtained by Sh. Coulon that investigated rolling friction (Figure 1 shows a diagram of the Coulon's experiment).

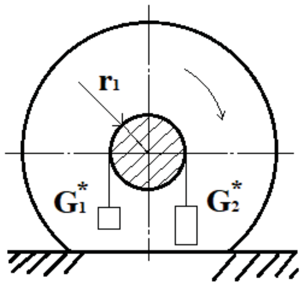

Figure 1. Coulon's scheme of rolling friction experience

Closer to reality is the model of a deformable wheel, which also contacts the deformable road in a certain area (multipoint contact). The O. Reynolds rod model is known [4], in which the wheel is represented as a set of elastic rods emanating from the common center of the wheel or thin disks on a single shaft. At any given time, the wheel contacts the road with several rods, and some rods slip while doing so, and some remain in constant contact. A similar effect occurs for the disk model.

For such models, O. Reynolds in 1876 drew attention to the effect of longitudinal pseudo-sliding, which consisted in the fact that *the path traveled by the center of the wheel of the vehicle (locomotive) DID NOT COINCIDE with the product of the angle of rotation of the wheel by its rolling radius.* Further, in 1925, motorists (Brulier) discovered the phenomenon of lateral pseudo-sliding (withdrawal), which consisted in changing the trajectory of the car under the action of lateral forces (for curved movements) in comparison with what should have been for a car with absolutely solid wheels. Finally, the rolling theory, which takes into account the effects of longitudinal and lateral pseudo-sliding in the case of a railway wheel, was developed in 1926-28 by F.Carter.

Thus, when the deformable wheel is rolling, coupling zones and sliding zones appear in its contact area. Similar (flat) models for continuous (distributed) contact were considered in the works of A. Y. Ishlinskiy. At the same time, the classical Coulon'smodel was used to calculate the friction force (the main vector of tangential forces applied to the wheel), which was determined by the distribution of normal stresses in the contact area. Calculating the moments of distributed normal and tangential forces, A. Yu. Ishlinsky explained (both qualitatively and quantitatively) the origin of the moment of friction of rolling resistance. The main result of these studies is as follows. The rolling friction moment is determined by the distribution of normal reactions in the wheel contact area, and also depends on the accepted model for friction forces.

Further development of the dry friction model (for the wheel, in particular) was obtained in the works of N. E. Zhukovsky, M. A. Levin, N. A. Fufaev, Kontensu and V. F. Zhuravlev. In these works, dry friction models were developed based on the principle of summation (integration) of elementary friction forces, as well as their elementary moments. In addition, it is also necessary to consider various (theoretically and practically acceptable) laws of distribution of normal stresses in the contact area. It is these stresses that determine the elementary friction force of the Coulomb. In turn, the law of distribution of normal stresses is determined by the dynamics of vehicle movement (for example, this law depends on whether the vehicle is moving with acceleration or not). Thus, it turns out that the driving force of friction depends on the way the vehicle moves, and this dependence is mutual.

In the tasks of describing the dynamics of ground vehicles, great importance is given to non-holonomic dynamic systems. By the term "non-holonomic systems" we mean a class of nonlinear systems that cannot be coordinated by continuous time-invariant feedback, i.e., there are times when certain constraints are imposed on the state of the system (non-holonomic connections). These systems are controllable, but they cannot move in some directions instantly.

They belong to the class of nonlinear differential systems with non-integrable constraints on motion. Non-holonomic control systems resulting from the formalization of non-holonomic systems include control inputs, are nonlinear control tasks requiring nonlinear tuning. Non-holonomic control systems are being actively studied in connection with the development of robots, including mobile robots, wheeled vehicles and space robotics, etc. In the case of wheeled vehicles, kinematics and dynamics can be modeled based on the assumption that the wheels roll perfectly (Fig. 2).

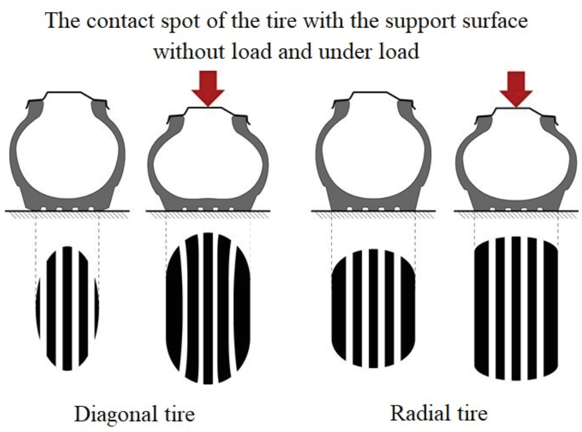

Figure 2. The contact spot of the tire with the support surface without load and under load

Typical limitations of wheeled vehicles are rolling. contact, such as rolling wheels on the ground without slipping, or sliding contact, such as skates sliding.

1. The spinning top model

For the most part, the spinning top was driven into rotation using a thin string previously wound on its shaft. Quickly pulling the string from the shaft of the spinning top, the latter was informed of rotation around the axis AA, 1 which lasted until the friction forces acting at the point

About the supports of the spinning top on any base, did not stop its movement. Many scientists of the world have taken up the study of the laws of motion of the spinning top. The famous English scientist I. Newton (1642-1727) and a member of the Russian Academy of Sciences L. Euler (1707-1783) also worked on this task. Euler in 1765 for the first time published the theory of motion of a solid body near a fixed point of its support and thereby created a theoretical basis for further in-depth study of the laws of motion of the spinning top. The works of French scientists J. Lagrange (1736-1813) and L. Poinsot (1777-1859) greatly contributed to the further study and development of methods for the practical use of the properties of a rapidly rotating spinning top.

In 1886, the French Admiral Fleurieu proposed a new device – sextant – for measuring the geographical latitude of the ship's location during a storm, the basis of which was a rapidly rotating spinning top. The spinning top itself was made in the form of a cylindrical body B (Fig. 3), supported by a pointed hairpin at point N. During operation, the device was held by the handle R in an upright position. With the help of a hand pump, compressed air was pumped into it through the hose M, which hit the side surface of the spinning top with directed jets and caused it to rotate around the AAX axis. With the weight of the spinning top at 175 g, it was possible to inform it of rotation at a speed of about 3000 rpm. To ensure the rotation of the spinning top invariably in the horizontal plane, its center of gravity was positioned approximately 1 mm below the fulcrum. The spinning top, even when the handle deviated from the vertical position, continued to remain in the horizontal plane, providing an artificial horizon on the rocking ship.

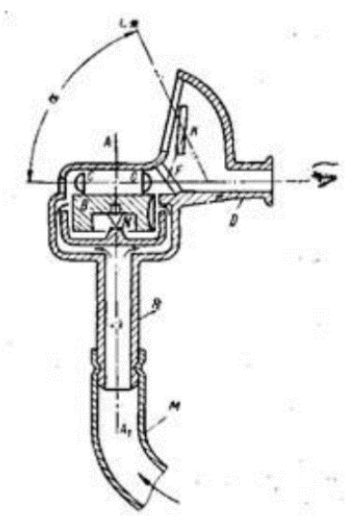

Figure 3. The spinning top model

For the convenience of fixing the horizon plane on the upper end surface of the spinning top, two plano-convex lenses C were fixed, on the flat surfaces of which thin strokes were applied, located parallel to the end surface of the spinning top. The distance between the lenses C corresponded to the focal length, as a result of which, when the spinning top was rotated, the strokes applied to the lenses for the eye observing through the eyepiece D of the device merged into one line. This feature fixed the position of the horizon plane, with respect to which the angle was measured, and the height of the luminary L, similar to how it was described above (Fig. 3).

For simultaneous observation of the artificial horizon line and the luminary, two mirrors A and K were installed in the device. To the turns of the mirror K, the beam coming from the luminary L was combined with the line of the artificial horizon. In this case, the magnitude of the angle *a* was determined by the angle of rotation of the mirror K. This device is considered to be the first invention in which a spinning top was used, in its shape and device not fundamentally different from ordinary spinning tops, which were widely used in everyday life.

Imagine a spinning top, for example, a thin brass disc (gear) mounted on a thin steel axle. The dynamics of such a spinning top generates the occurrence of precession, gyroscopic moment and other characteristics of the movement of the spinning top.

We introduce a unit vector , showing the direction of the axis of a symmetrical spinning top in space, i.e., coming from the origin of the coordinate system (from the center of mass) and directed along the axis of the spinning top (Fig. 4).

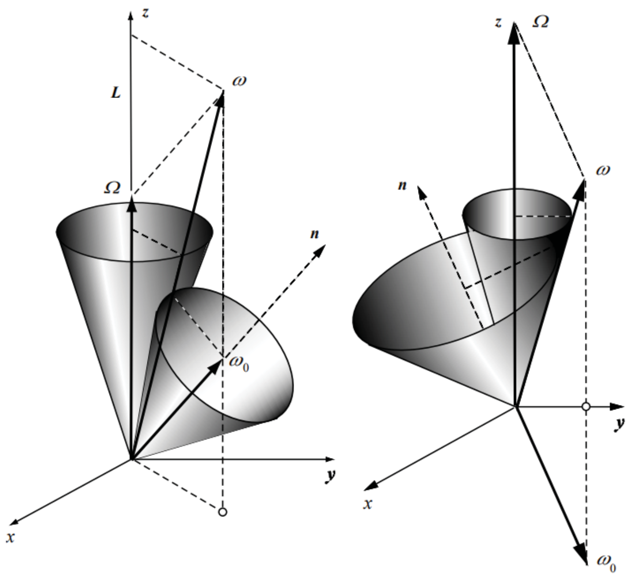

Figure 4. Geometric interpretation of the free precession of a symmetric spinning top

Let – the moment of the pulse, and – angular velocity vector. At each moment of time, all three vectors and they lie in the same plane, and when the body moves, their relative position remains unchanged. In the absence of moments of external forces, a plane containing vectors и , rotates uniformly around the direction of the vector unchanged in space . In fact, the speed the point of the spinning top axis that coincides with the end of the vector *n*, expressed in terms of angular velocity by the formula .

This means that at any moment the end of the vector n it moves perpendicular to the plane under consideration, dragging it along with the vectors lying in it n and ω. Thus, the entire plane rotates uniformly around$L$, and the vectors lying in it n and ω synchronously describe cones in space, the vertices of which lie at the origin. About this behavior of vectors n and ω they say they commit around L *regular precession*. It can be shown that the angular velocity of this precession Ω proportional to the moment of the pulse$L$ and is inversely proportional to the central moment of inertia of the spinning top $I \perp$ relative to the transverse axis: $\Omega = L/I \perp$. Such a free precession of the axis of the spinning top, which occurs in the absence of external moments when the angular velocity does not coincide with the axis of the spinning top, is also called nutation. Note that the axis of the spinning top retains its direction in space (does not precess) if, during free rotation, the angular velocity is directed along the axis of the spinning top, i.e., in such cases nutation does not occur.

2. From gyroscope to multicomponent dry friction

As a result of experimental observations of the behavior of the Fleurieu gyroscope, which Contence conducted, he began his research, as he received evidence of unsatisfactory compliance with the theory. Theoretical predictions initially followed from the ideas about the interaction of the gyroscope support with the base axle either in the form of one-dimensional dry friction, or in the form of no slippage at the point of contact (non-holonomic formulation), or in the form of purely viscous friction.

Contensu noted [15] that the use of Coulomb's law to describe friction in the case of a combination of simple movements (sliding and rolling, sliding and twisting) should not be true. This is evidenced by simple and well-known experiments, for example, rotation around the vertical of a car wheel when rolling or the sliding of a polisher brush when it rotates. Although the contact zone of the rubbing bodies in the case of the Fleurieu gyroscope was negligible and many simply considered the contact point, Contensu called this zone a circle and considered Coulomb's law in differential form inside this circle so.

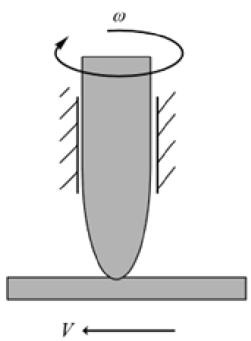

Figure 5. Rotating rod

We will consider a vertically rotating rod resting with a spherical end on a flat support moving at a constant speed (Fig. 5.). The contact area is a circle of radius ε, in which the normal voltage depends only on the distance ρ to the center of the circle: $\sigma(\rho)$ Relative sliding is carried out at a speed of v, the angular velocity of the spin is indicated by ω (Fig. 6).

Relative sliding speed v_c at a point having polar coordinates in the contact area ρ, θ it is expressed as follows:

$$v = (v - \omega\rho\sin\theta, \omega\rho\cos\theta).$$

The differential of the friction force directed against the relative velocity at this point, in accordance with Coulomb's law, has the form:

$$dF = -f\,\sigma(\rho)\frac{v_c}{|v_c|}ds.$$

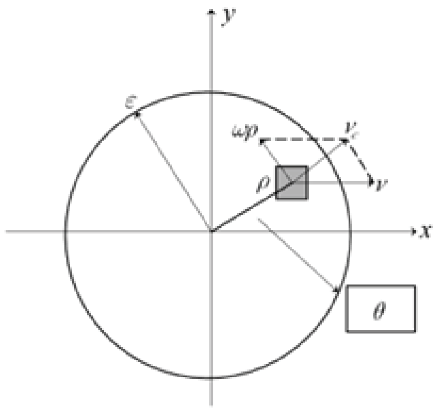

Figure 6. Contact area

The moment of this elementary force:

$$dM = -|\rho \times dF| = -\rho_x dF_y + \rho_y dF_x$$
$$= f\,\sigma(\rho)\,\rho(\rho\omega - v\sin\theta)\frac{ds}{|v_c|}.$$

As a result, for the moment and force we get the following expressions:

$$M = -f\int_0^{\varepsilon}\int_0^{2\pi}\frac{(\rho\omega - v\sin\theta)\sigma(\rho)\rho^2 d\rho d\theta}{\sqrt{\omega^2\rho^2 - 2v\omega\rho\sin\theta + v^2}},$$

$$F= M = -f\int_0^{\varepsilon}\int_0^{2\pi}\frac{(v - \omega\rho\sin\theta, \omega\rho\cos\theta)\sigma(\rho)\rho^2 d\rho d\theta}{\sqrt{\omega^2\rho^2 - 2v\omega\rho\sin\theta + v^2}}.$$

Note that due to the symmetry, the expressions for the force relative to the x-axis of its component along the y-axis are zero.

Let 's introduce the notation $u = \omega\varepsilon, r = \rho/\varepsilon$. Given these notations, we rewrite the expressions for the modulus of the moment and the modulus of the nonzero component of the force:

$$M(u,v) = f\varepsilon^3\int_0^1 r\sigma(r)\int_0^{2\pi}\frac{ur^2 - vr\sin\theta}{\sqrt{u^2r^2 - 2uvr\sin\theta + v^2}}d\theta dr,$$

$$F(u,v) = f\varepsilon^2\int_0^1 r\sigma(r)\int_0^{2\pi}\frac{v - ur\sin\theta}{\sqrt{u^2r^2 - 2uvr\sin\theta + v^2}}d\theta dr.$$

Let us first consider as an example a point contact by Hertz. We assume that both contacting surfaces are locally spherical, then:

$$\sigma(r) = \frac{3N}{2\pi\varepsilon^2}\sqrt{1 - r^2}\,.$$

From the presented relations it follows that when ε⇒0 *the moment of friction also tends to zero, and for this reason it was not even considered. As for the friction force, it has the form:*

$$F(u,v) = f\frac{3fN}{2\pi}\int_0^1 r\sqrt{1 - r^2}\int_0^{2\pi}\frac{(v - ur\sin\theta)d\theta dr}{\sqrt{u^2r^2 - 2uvr\sin\theta + v^2}}.$$

Argument $u = \omega\varepsilon$ has the order of smallness ε at the end ω. In a small area $|v| \leq |u|$ this function can be approximated by a tangent at zero, and outside of it by a horizontal line corresponding to the usual Coulon's law.

The obtained ratio allows us to draw a fundamentally important conclusion: in the case of point contact, the friction force does not have the form of the original Coulomb law, in particular, it is equal to zero identically by u: $F(0, u) \equiv 0$, $(u \neq 0)$. That is, there is no resting friction force, if only the twisting takes place, no matter how small it may be. So, references to the law of dry friction to justify the absence of slippage when using a non-holonomic formulation of the problem of rolling some bodies by others, as is often done, are inappropriate.

The dependence of the friction force at a point on the spinning speed is very significant and there is no way to neglect it. In cases where researchers in problems with combined friction accept the condition of no slippage, they must either indicate by what physical forces it can be provided, or, realizing such a condition is approximate, talk about evaluating the accuracy of such an approximation. The term "absolutely rough surface" cannot be based on the idea of dry friction.

3. Rolling theory approaches

In addition to sliding friction, Sh. Coulon studied rolling friction, for which he created an experimental setup (Fig. 7) consisting of two parallel wooden bars on which a cylindrical wooden roller rolled. A moment proportional to the difference in the weights of the loads fixed at opposite ends of the rope was applied to the roller by means of a rope thrown over it. The results of experiments on this installation Coulomb expressed the widely used and currently used formula for the force overcoming the rolling resistance of the roller:

$$T = k\frac{Q}{r},$$

where *Q* – the weight of the roller together with the loads, *r* – radius of the rink, *k* – the proportionality coefficient having the dimension of length.

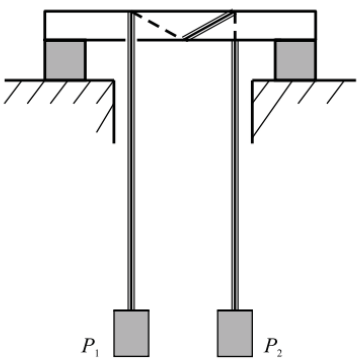

Figure 7. Coulon's experimental setup

Similarly, to the sliding friction law, in a more detailed entry, the rolling friction force has the form:

$$T = \begin{cases} k\frac{Q}{r}\, sgn\, \omega, & \omega = 0, \\ \left[-k\frac{Q}{r}, k\frac{Q}{r}\right], & \omega = 0, \end{cases}$$

where ω represents the angular velocity of the roller. If there is no rolling, then the rolling force can take any value in the specified interval.

Rolling friction is a more complex phenomenon than sliding friction. In the case of sliding, the contact area is stationary relative to the sliding body, in the case of rolling, this area is movable both relative to the body and relative to the stationary surface. In addition, rolling cannot occur without sliding friction.

Let the center of the rink be carried away by force *T*, attached to its center, moving at a constant speed *V* (Fig. 8). It is required to find the law of changing the angle of rotation of the roller.

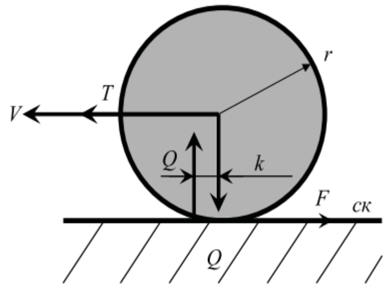

Figure 8. The scheme of the skating rink movement

Let 's write down the equations of motion of the rink

$$m\dot{V} = T - F_{ск}^{\cdot} = T - fQ,$$
$$J\dot{\omega} = -M_{кач} + rF_{ск} = -kQ + rF_{ск}$$

(We assume for certainty that $\omega \geq 0, V - \omega r \geq 0$).

If the velocity of the center of mass is constant $(\dot{V} \equiv 0)$, that $T = fQ$ *and* the second equation of the written system takes the form:

$$J\dot{\omega} = (-k + fr)Q.$$

If $fr > k$ (sliding friction prevails over rolling friction), then the rotation of the roller accelerates until the slipping of the roller relative to the base stops. The sliding friction force becomes significant at the point of rupture: $Q < F_{ск} = k\frac{Q}{r} < fQ.$

If $fr < k$ (rolling friction prevails over sliding friction), then the rotation of the roller slows down, the limiting movement of the roller is translational, the sliding speed of the roller relative to the base is equal to the speed of the center of the rink. The rolling friction moment becomes significant at the point of rupture: $0 < M_{кач} = fQr < kQ$.

If $fr = k$, then the angular velocity does not change its magnitude, both rolling and sliding take place in stationary mode.

It is known that mankind invented the wheel as a means of overcoming dry friction. In the case of rolling an absolutely rigid wheel on an absolutely rigid horizontal surface, there is really no friction, i.e., slippage. In reality, energy losses during rolling remain, although they become significantly less.

A significant contribution was made to the construction of a qualitative rolling theory in 1876. Osborne Reynolds, [4], who discovered the following experimental fact. It consists in. That the area of contact of elastic bodies during rolling is divided into areas of sliding (slipping) and adhesion (setting), Figure 9. At the same time, with an increase in the moving or braking moment, the area of the slip zone increases, and the area of the adhesion zone decreases.

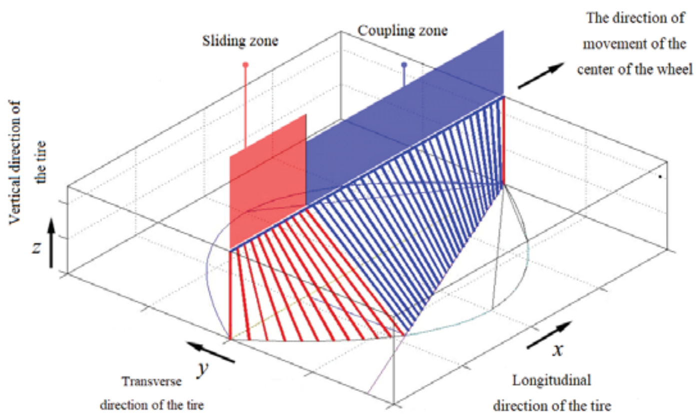

Figure 9. Diagram of zones in the tire contact spot with the road

This happens until the area of the coupling zone becomes zero and a complete slip occurs. The zones are characterized as follows. In the slip zone, the points of contact between the wheels and the road move relative to each other, and in the clutch zone these points are stationary relative to each other and the wheel is rolling.

4. Classical examples are Chaplygin's sleigh and the Appel mechanism

In [5], the problem of controlling the movement of three-wheeled robots with two driving and passive piano wheels is considered. The generated model is reduced to a system:

$$\omega^* - e\omega^* - a\omega^2 + \sigma v + \gamma\sigma\omega = p,$$
$$(J + \theta e^2)\omega^* + av\omega - ea\omega^2 + \sigma\omega + \gamma\sigma v + e\sigma v + e\gamma\sigma\omega = q + ep \qquad (1)$$

Here are the parameters $J > 0$, $\theta > 0$ determined by the inertial mass characteristics of the system, a, e set the position of the center of mass of the body relative to the wheels, $\sigma \geq 0$ – normalized viscous friction in wheel axles, $\gamma(|\gamma| < 1)$ – parameter that determines the asymmetry of friction, p, q – co rol of the longitudinal speed and rotation of the housing. It is assumed $a > 0$, since the reverse situation is equivalent to changing the sign of the speed.

The behavior of the system is investigated in the case when the control signals p, q permanent. Then the system (1) is autonomous and the use of the phase plane ω, v it is very convenient for studying movements at different parameter values.

Simple cases. At certain values of the parameters, equations (1) turn into equations of nonholonomic systems considered by the classics in the works.

Chaplygin's Sleigh. When $p = q = e = \sigma = \gamma = 0$ system (1) takes the form:

$$v^* - a\omega^2 = 0, \; J\omega^* + av\omega = 0, \qquad (2)$$

coincides with the system obtained and studied by Chaplygin [11] and Karateodori [12], who studied the inertia motion along the horizontal plane of the "Chaplygin sleigh – a nonholonomic mechanical system representing a solid body resting on the plane with two "slippery" points and the point of the skate blade. The position of the contact point of the skate corresponds to the middle of the segment connecting the attachment points of the wheels of the mobile robot. Stationary points $\omega_0 = 0, \; v_0 = const$ systems (2) fill the entire ordinate axis. It is obvious that the stationary points correspond to the robot's movements at a constant speed along straight line.

System (2) has an integral:

$$v^2 + J\omega^2 = const,$$

which defines a family of ellipses $(J > 0)$ – pha trajectories on the plane ω, v. When $a > 0$ the image point moves along a phase trajectory from bottom to top; hence, stationary points $v_0 < 0$ unstable, $v_0 > 0$ – able. Thus, the movements of the robot with the center of mass of the body behind the wheels are unstable, in front - stable. The intersection of the phase trajectory of the abscissa axis corresponds to the point of return of the trajectory of the robot.

Appeal Mechanism. When $q = e = \sigma = \gamma = 0$ system (1) takes the form:

$$v^* - a\omega^2 = p, \; J\omega^* + av\omega = 0. \qquad (3)$$

These equations coincide with the equations obtained and studied by Appel [13] and later by Hamel [14] for a nonholonomic mechanical system, which differs from Chaplygin's sleigh in that it has a wheel instead of a skate, which is affected by a constant torque created by means of a load on a thread thrown over a block mounted on the body and wound on a pulley coaxial with the wheel. Here are some results of these works. If $p > 0$, then there are no stationary points; if $p < 0$, then we have two stationary points:

$$v_0 = 0, \; \omega_0 = \pm\sqrt{-p/a}. \qquad (4)$$

Autonomous equations (3), excluding time, can be reduced to the form

$$(p/\omega^2 + a)d(\omega^2) = -(a/J)d(v^2). \qquad (5)$$

Equation (5) obviously has an integral

$$a\omega^2 + p\ln(\omega^2) + (a/J)v^2 = C,$$

where C – an arbitrary constant. Phase trajectories for the case $p < 0$ are shown in Fig. 10. The stationary points in this case are the centers. It can be seen that stationary motion (4) – the rotation of the body at a constant speed relative to a non-moving point – is realized under the condition that the moment p on the wheel balances the centrifugal force $a\omega^2$.

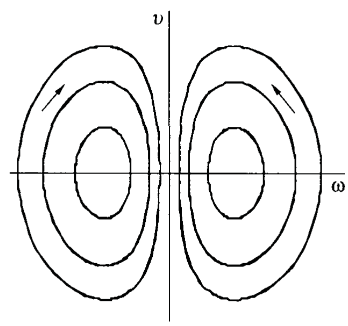

Figure 10. Phase trajectories of the "Appel Mechanism"

5. Connection of dry friction, rolling and contact spots

The forces acting on the tires of a ground vehicle determine the trajectory or path of the vehicle. These forces are limited by the friction of the contact spot between the tire and the road surface.

The ability to obtain information about the value of the coefficient of friction before using maximum tire forces can be a determining factor in preventing an accident or in significantly mitigating the severity of the situation. It is known that in practice all chassis safety is provided by systems such as electronic stability control and anti-lock braking system and more modern systems that mitigate the consequences of collisions by braking. The efficiency of such systems can be significantly improved if information about the current level of friction is sent to the control systems.

Tire modeling is an important step in the process of understanding and evaluating the friction and force components acting in the contact spot between the tire and the road. Research on tire characteristics and modeling of their dynamics has been actively conducted over the past 70 years. The level of detail in these models ranges from basic first-order local effects at the macroscopic scale to detailed high-level models at the microscopic scale, for example, described by finite element methods.

The classical representation of the dynamics of the wheel of a ground vehicle in modern engineering is a tire model that takes into account the phenomenon of force sliding in the contact spot on a macroscopic scale. Practical applications of complex theoretical models of mechanics require simplified, basic relations, so it is sufficient to consider stationary steady-state tire models where the parameters take constant values. Then the onboard sensors will not display fast stochastic and poorly controlled signals taken from the bus zones.

Another aspect related to the choice of a model is its models and, in particular, complexity in terms of the number of parameters. It is well known that excessive parameterization in models leads to a lack of convergence of model parameters or to incorrect modeling results. Due to the complexity of interaction and the presence of random factors of road infrastructure, this fact is especially evident in the problems of estimating friction parameters in real automobile traffic. Therefore, models with a minimal set of parameters describing the dynamics of the wheel have a high priority for technical applications.

Most adequate tire models are tied to the properties of power components that provide interaction with sliding and rolling forces. It is worth noting the dependence discovered by Bakker [9] and then developed by Pacejka [8], which is now commonly called the "magic formula".

The general form of the Magic Formula, given by Pacejka [8], [9], is:

$$y = D\,sin\{C\,arctan\,[Bx - E\,(arctan\,(Bx))]\},$$

where B, C, D and E represent fitting constants and y is a force or moment resulting from a slip parameter x. The formula may be translated away from the origin of the x–y axes. The Magic Model became the basis for many variants.

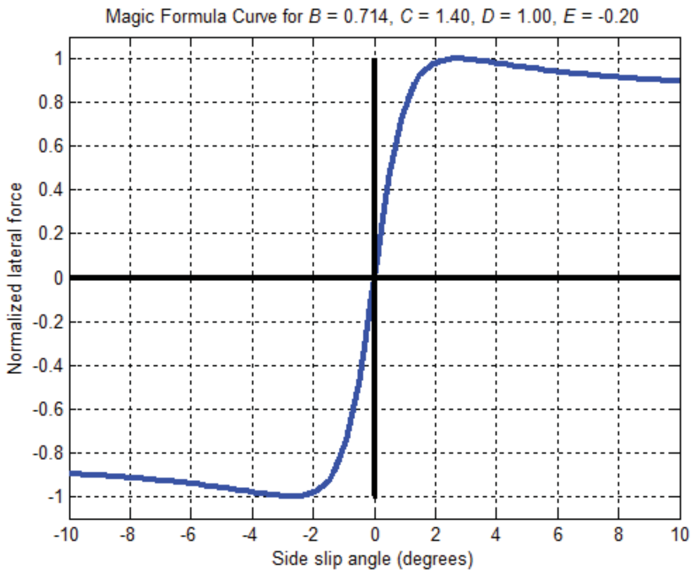

Figure 11. Pacejka's graph of function dependence from Magic Formula

An interesting application of this formula for the design of multi-wheeled mobile robots is proposed in [10].

The most productive for applications is the brush, brush model (Brush model), the advantages of which are that it allows you to describe the evolution of force components in the contact spot.

Swedish researchers, namely the well-known scientific group of Svendenius, [3], proposed a simple version of the model of a clean sliding brush (Brush model). This model has been confirmed by experiments on real data, and allows it to be adapted to different road surfaces and different types of tires.

The advantages of the brush model are that only some simple assumptions about the properties of the tire, the contact spot and the characteristics of the road surface are enough to formalize it.

We formulate standard assumptions that allow us to obtain a model that is completely determined by two parameters. The basic assumption is that the tire can be divided into an infinite number of bristles that deflect when in contact with the road surface. Each bristle stretches in the transverse direction, is considered to have an elastic reaction and deforms independently of other bristles. The spot of contact with the bristles is additionally illustrated in figure 12.

In addition, standard assumptions are introduced.

1. The vertical distribution of tire pressure is a parabolic function.
2. The friction force between the tire and the road is described by Coulomb friction, i.e., there is a pre-shear effect, etc.
3. The friction force is considered isotropic, i.e., the friction force is limited to a circle in the plane of the road.
4. The influence of the camber angle is not taken into account.
5. It is assumed that the tire frame is rigid, and it is assumed that all flexible movements occur in the bristle in the contact spot.

The road surface is considered flat and solid, i.e., no part of the road surface is moved or transported in a contact spot, for example, on snow-covered roads or gravel roads.

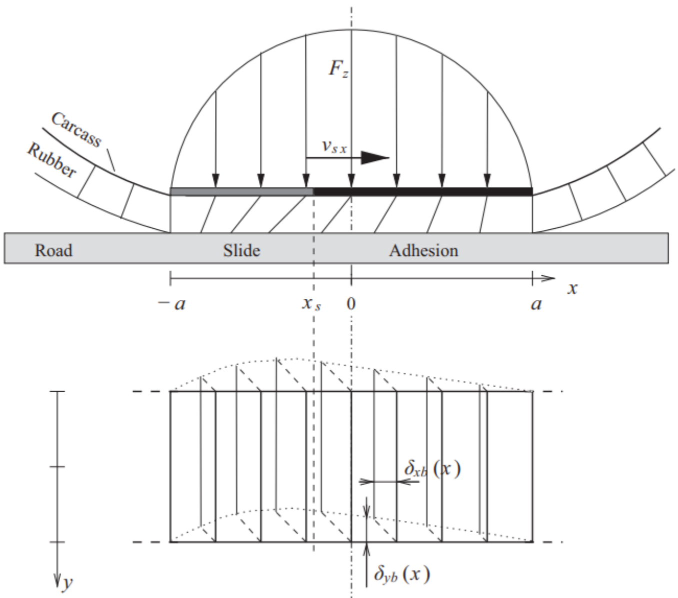

Figure 12. Scheme of brush model bristles in the contact spot

Note that Assumption 1 is standard, and the arguments in favor of this assumption are confirmed by experimental data. Moreover, it is valid both for the case of pure longitudinal sliding and for the case of pure lateral sliding.

Assumption 2 reduces the number of parameters, so that only one parameter related to friction is sufficient necessary.

Assumption 3 allows us to characterize the tire friction in both the longitudinal and transverse directions, and with one parameter. This is also a simplification of the tire model of a real ground vehicle, which should not have a frame and the same thread pattern in the longitudinal and transverse directions. The assumption made is a compromise between simplicity and accuracy.

Assumption 4 does not take into account the camber angle is also introduced to simplify the model, it is assumed that the camber angle is 0. If we do not make this assumption, we will also get an additional parameter.

Assumption 5 is the standard assumption for the brush approach and has higher confidence in the longitudinal direction. The assumption is not to include the rigidity of the frame also to minimize the number of parameters.

Assumption 6 on the road surface makes it possible to simplify the description of the interaction of the tire with the road with fewer parameters.

Conclusion

In this paper, we present an approach that allows us to analyze the processes of wheel dynamics during the movement of a ground vehicle. Comparisons of various elastic wheel models are carried out within the framework of the theory of multicomponent dry friction. The ways of theory development for practical engineering applications are outlined.

References

1. V.V. Kozlov (2010). Lagranzheva mekhanika i sukhoe trenie [Lagrangian mechanics and dry friction]. *Nelineinaya dinamika* [Nonlinear dynamics]. Vol. 6, No. 4, pp. 855-868.
2. A.A. Andronov, V.F. Zhuravlev (2010). Sukhoe trenie v zadachakh mekhaniki [Dry friction in problems of mechanics]. Moscow: Regulyarnaya i khaoticheskaya dinamika Publ.; Izhevsk: In-t komp'yut. issled. Publ. 184 p.
3. V.V. Andronov, V.F. Zhuravlev (2010). Dry friction in problems of mechanics. Moscow – Izhevsk: Research Center "Regular and Chaotic Dynamics", Institute of Computer Research. 184 p.
4. J. Svendenius (2003). Tire Models for Use in Braking Applications. Department of Automatic Control Lund Institute of Technology. Lund, 2003. 95 p.
5. O. Rynolds (1876). On rolling friction. *Philosophical Transactions of the Royal Society of London.* Vol. 166 (I). Pp. 155-174.
6. F. Bruzelius, M. Hjort, J. Svendenius (2014). Validation of a basic combined-slip tyre model for use in friction estimation applications. *Proc IMechE Part D: J Automobile Engineering 2014*, Vol. 228(13). Pp. 1622-1629. DOI: 10.1177/0954407013511797.
7. V.M. Budanov, E.A. Devyanin. On the movement of wheeled robots. *Applied Mathematics and Mechanics*, 67 (2), pp. 244-244.
8. V.A. Koronatov (2018). Presentation of a new theory of rolling on the example of describing the movement of the driven wheel of a locomotive (car). *Modern technologies. System analysis. Modeling*, (1 (57)), pp. 49-59.
9. G. Rosenblat, V. Tishkin, V. Yashin (2020). Model of carriage movement on plane with dry friction forces. *iJOE*. Vol. 16, No. 8, pp. 85-89.
10. H.B. Pacejka (2012). Tire and vehicle dynamics. Besselink, Igo (3rd ed.). Oxford: Butterworth-Heinemann. p. 165. ISBN 978-0-08-097016-5. OCLC 785829133.
11. E. Bakker, H.B. Pacejka, L. Lidner. (1989). A New Tire Model with Application in Vehicle Dynamics Studies. *Proc. 4th Int. Conf. Automotive Technologies, Monte Carlo*, SAE paper 890087.
12. A.P. Aliseichik, V.E. Pavlovsky (2010). Research methodology for dynamic comfort of movement of a multi-wheeled mobile robot. *M.V. Keldysh Institute preprints*. No. 84.27
13. S.A. Chaplygin (1911). On the theory of motion of nonholonomic systems, the reducing factor theorem. *Mathematical collection*. Vol. 28, no. 2, pp. 303-314.
14. C. Carathéodory (1933). Der schlitten. ZAMM. *Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, 13(2), 71-76.
15. M.P. Appell (1911). Exemple de mouvement d'un point assujetti é une liaison exprimée par une relation non linéaire entre les composantes de la vitesse. *Rendiconti del Circolo Matematico di Palermo* (1884-1940), 32(1), 48-50.
16. G. Hamel (1978). Theoretische Mechanik: Eine einheitliche Einf¨uhrung in die gesamte Mechanik. 2nd ed. Berlin: Springer. 796 p.
17. P. Contensu (1967). The relationship between sliding friction and spinning friction and its consideration in the theory of the top. Gyroscopy problems. Moscow: Mir. Pp. 60-77.

МОДЕЛИ АВТОМОБИЛЬНОГО КОЛЕСА, ИСПОЛЬЗУЮЩИЕ ЗАКОНЫ РАСПРЕДЕЛЕНИЯ СИЛОВЫХ ВОЗДЕЙСТВИЙ В ОБЛАСТИ КОНТАКТА С ДОРОЖНЫМ ПОКРЫТИЕМ

Карелина Мария Юрьевна, *Московский автомобильно-дорожный государственный технический университет (МАДИ), Москва, Россия,* **karelina@madi.ru**

Терентьев Алексей Вячеславович, *Московский автомобильно-дорожный государственный технический университет (МАДИ), Москва, Россия*

Розенблат Григорий Маркович, *Московский автомобильно-дорожный государственный технический университет (МАДИ), Москва, Россия,* **gr51@mail.ru**

Яшин Владимир, *Московский автомобильно-дорожный государственный технический университет (МАДИ), Москва, Россия,* **hekkoki@gmail.com**

Аннотация

Динамику качения автомобильного колеса естественно рассматривать при его взаимодействии с дорожным покрытием. При этом наиболее трудной и важной задачей является определение силовых компонентов, приложенных к колесу, таких как движущей силы трения, силы сопротивления, моментов сопротивления качению и верчению, которые возникают в пятне контакта со стороны полотна дороги. В работе исследованы аспекты сухого трения, качения и скольжения автомобильного колеса, представленного как деформируемое тело. В этом случае большое значение имеет учет протекторов, что отражается в моделях шин. Важным аспектом является изучение законов распределения нормальных напряжений в области контакта. Для решения практических вопросов автомобильного транспорта выделены подходы, основанные на Magic Formula Пасейки и методы расчетов, основанные на щеточных, ленточных и струнных моделях, в частности, Brush-модель Свендениуса. Получены и обоснованы условия ее применимости.

Ключевые слова: *Модель шины, модель щетки, сухое трение, пятно контакта.*

Литература

1. *Козлов В.В.* Лагранжева механика и сухое трение // Нелинейная динамика. 2010. Т. 6. № 4. С. 855-868.
2. *Андронов А.А., Журавлев В.Ф.* Сухое трение в задачах механики. М.-Ижевск: НИЦ "Регулярная и хаотическая динамика", Институт компьютерных исследований, 2010. 184 с.
3. *Svendenius J.* Tire Models for Use in Braking Applications // Department of Automatic Control Lund Institute of Technology. Lund, 2003. 95 p.
4. *Rynolds O.* On rolling friction // Philosophical Transactions of the Royal Society of London. 1876. Vol. 166 (I). Pp. 155-174.
5. *Bruzelius F., Hjort M., Svendenius J.* Validation of a basic combined-slip tyre model for use in friction estimation applications. Proc IMechE Part D: J Automobile Engineering 2014, Vol. 228(13) 1622-1629. DOI: 10.1177/0954407013511797.
6. *Буданов В.М., Девянин Е.А.* О движении колесных роботов. Прикладная математика и механика, 67(2), 244-244.
7. *Коронатов В.А.* Представление новой теории качения на примере описания движения ведомого колеса локомотива (автомобиля). Современные технологии. Системный анализ. Моделирование, 2018, (1 (57)). С. 49-59.
8. *Rosenblat G., Tishkin V., Yashin V.* Model of carriage movement on plane with dry friction forces. iJOE. Vol. 16, No. 8, 2020. C. 85-89.
9. *Pacejka H.B.* Tire and vehicle dynamics. Besselink, Igo (3rd ed.). Oxford: Butterworth-Heinemann. 2012. 165 p. ISBN 978-0-08-097016-5. OCLC 785829133.
10. *Bakker E., Pacejka H.B., Lidner* L. A New Tire Model with Application in Vehicle Dynamics Studies. Proc. 4th Int. Conf. Automotive Technologies, Monte Carlo, 1989, SAE paper 890087, 1989.
11. *Алисейчик А.П., Павловский В.Е.* Методика исследования динамической комфортабельности движения многоколесного мобильного робота // Препринты ИПМ им. М.В.Келдыша. 2010. № 84. 27 с.
23. *Чаплыгин С.А.* К теории движения неголономных систем, теорема о приводящем множителе // Математический сборник. 1911, Т.28, вып. 2. С. 303-314.
13. *Carathodory C.* Der schlitten. ZAMM Journal of Applied Mathematics and Mechanics/Zeitschrift fur Angewandte Mathematik und Mechanik, 1933. 13(2), 71-76.
14. *Appell M.P.* (1911). Exemple de mouvement d'un point assujetti une liaison exprime par une relation non lin?aire entre les composantes de la vitesse. Rendiconti del Circolo Matematico di Palermo (1884-1940), 32(1), 48-50.
15. *Hamel G.* Theoretische Mechanik: Eine einheitliche Einf?uhrung in die gesamte Mechanik. 2nd ed. Berlin: Springer, 1978. 796 p.
16. *Контенсу П.* Связь между трением скольжения и трением верчения и ее учет в теории волчка. Проблемы гироскопии. М.: Мир 1967. С. 60-77.